Microwave Undulator II – Corrugated waveguide

Muralidhar Yeddulla

BROOKHAVEN NATIONAL LABORATORY

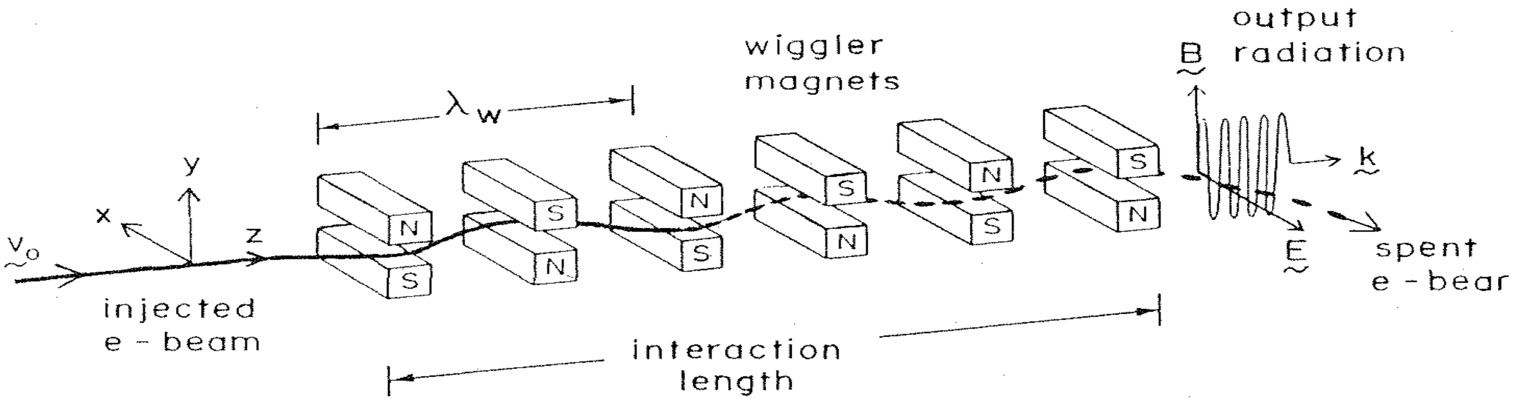
Sami Tantawi

SLAC NATIONAL ACCELERATOR LABORATORY

Acknowledgements

- The idea of using corrugated waveguides for RF undulators was first proposed by Dr. Sami Tantawi
- The work presented in this talk was done at SLAC by me and Dr. Tantawi

Physics of Undulator



- Highly relativistic electron beam passing through a pump field (wiggler field) produces synchrotron radiation
- Usually constructed from static periodic magnets
- Main undulator parameters are the period and K
  - K typically 2 to 3 for static undulators
  - Typical λu for existing static undulators few cm

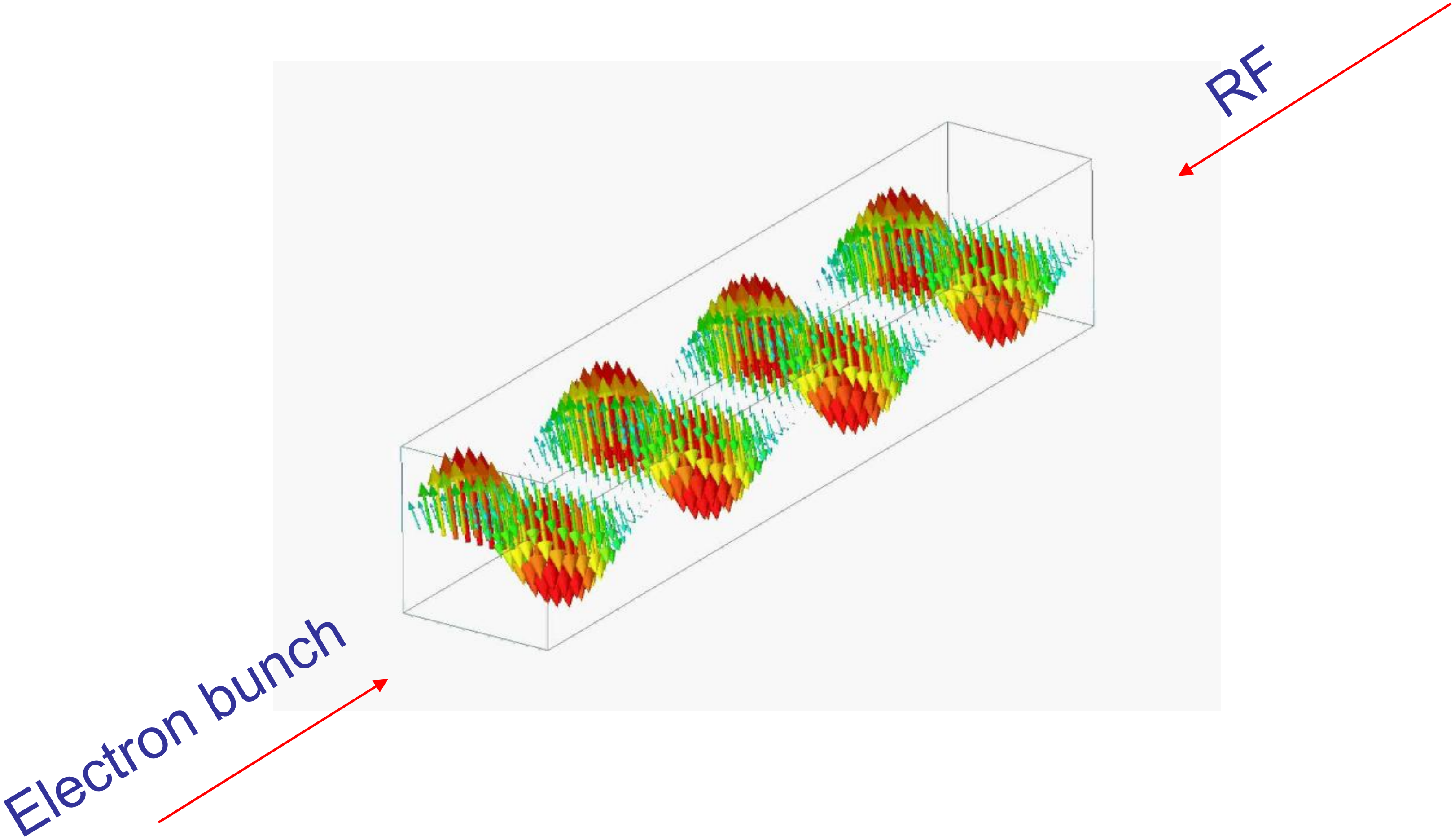
Why microwave undulator

- Limitations of permanent magnet undulators
  - Polarization cannot be controlled
  - Undulator period cannot be changed
- Advantages of a microwave undulator
  - No magnets to be damaged by radiation
  - Small undulator periods and larger apertures possible
  - Beams with larger radius and emittance can be used
  - Helical undulators are relatively easy with microwaves
- Drawbacks of a microwave undulator
  - High power microwave sources with **precise and stable amplitude and phase** are expensive
  - Handling of tens of GW of microwave power is challenging
  - Design of waveguide/cavity structure can be complicated

Applications of polarization control

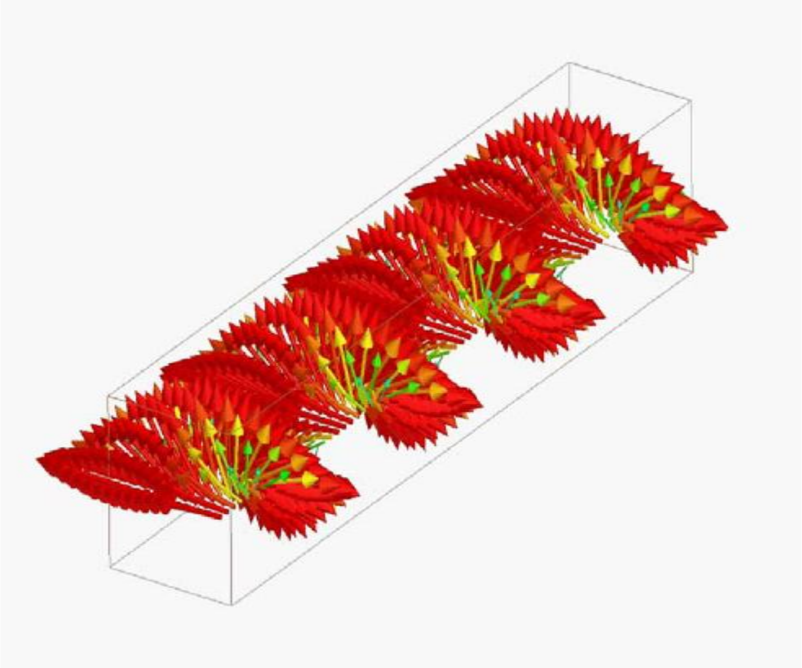
- Exciting scientific opportunity in areas where scattered or absorbed x-ray signal from sample depends on polarization state
- For example, measurement of very small magnetic moment changes in magnetic devices requires fast modulation and lock-in techniques to suppress systemic errors caused by slow drifts which is not possible with pump-probe techniques

Planar Microwave Undulator

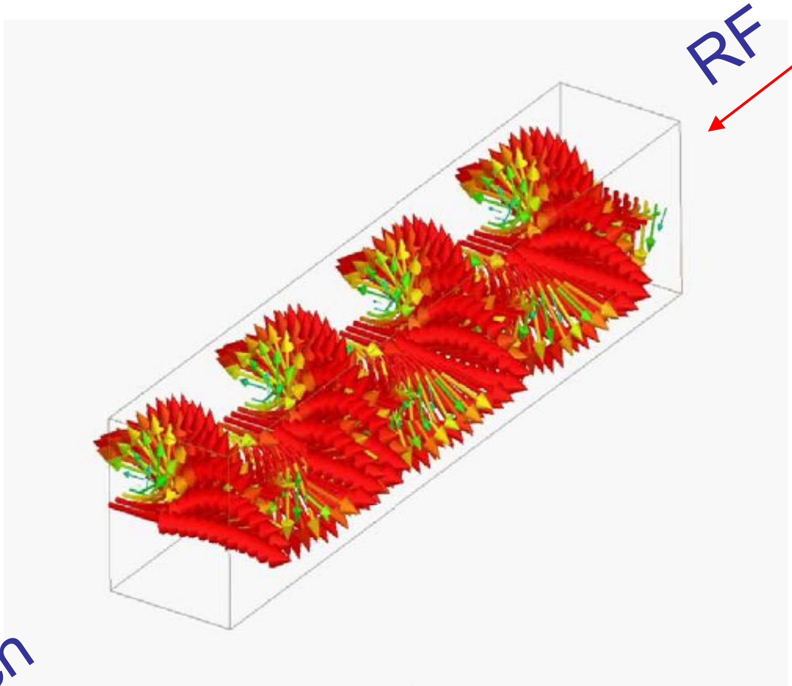


Vector plot of deflecting field in mid-plane of the waveguide

Helical Microwave Undulator



Right



RF

Left

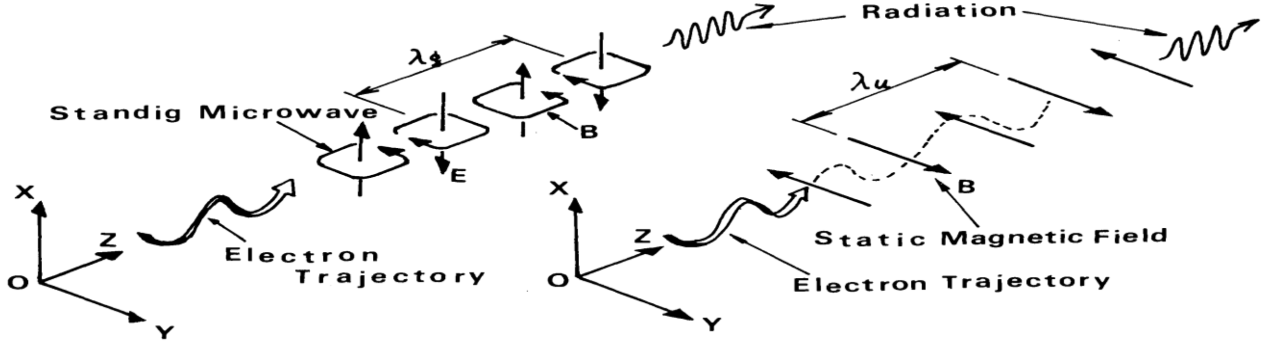
Electron bunch

Vector plot of deflecting field in mid-plane of the waveguide

Microwave undulator realization

- High power handling capabilities of the undulator waveguide have to be maximized and losses in the waveguide minimized
- The operating microwave mode should have maximum field strength in the path of electron beam and minimum tangential RF magnetic field near waveguide wall

Structure of microwave undulator

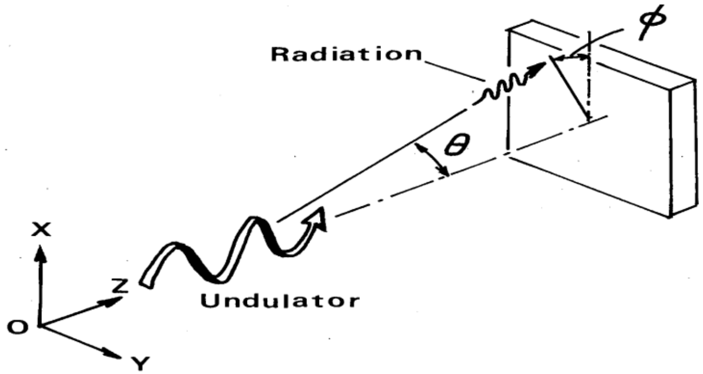


(a) Microwave Undulator (b) Magnetic Undulator

$$\frac{d\beta_x}{dt} = \frac{eB_u}{m_o\gamma}\cos\frac{2\pi z}{\lambda_u}$$ ← Eqn. of motion for magnetic undulator

$$\frac{d\beta_x}{dt} = -\frac{e}{m_o c\gamma}(E_x - v_z B_y)$$ ← Eqn. of motion for microwave undulator

Radiation Characteristics of Undulator



Radiation wave number

Helical Undulator

$$k_s = \frac{2\gamma^2}{1+2K^2+\gamma^2\theta^2} k_u$$

Planar Undulator

$$k_s = \frac{2\gamma^2}{1+\frac{K^2}{2}+\gamma^2\theta^2} k_u$$

Undulator parameter ⟶ $K = \frac{eB_u\lambda_u}{2\pi m_0 c}$

Undulator wave number = k_u

SLAC NATIONAL ACCELERATOR LABORATORY

BROOKHAVEN NATIONAL LABORATORY

Need for microwave cavity

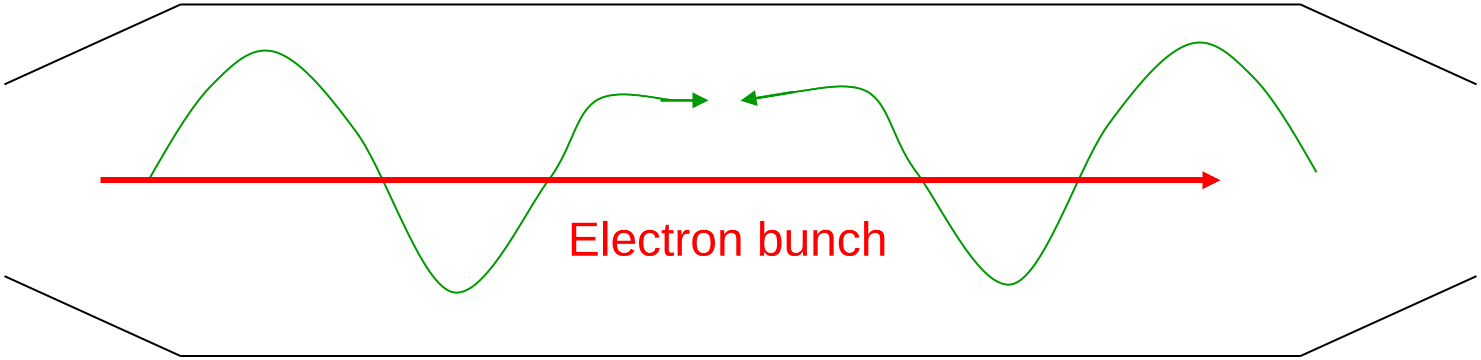
Magnetic field in conventional static undulator: $B_y = B_u \cos \frac{2\pi z}{\lambda_u}$

- To achieve comparable field strength B_y, tens of GW of RF power is needed.

- RF energy can be stored in a cavity by pumping in RF power to achieve the required field strength B_y

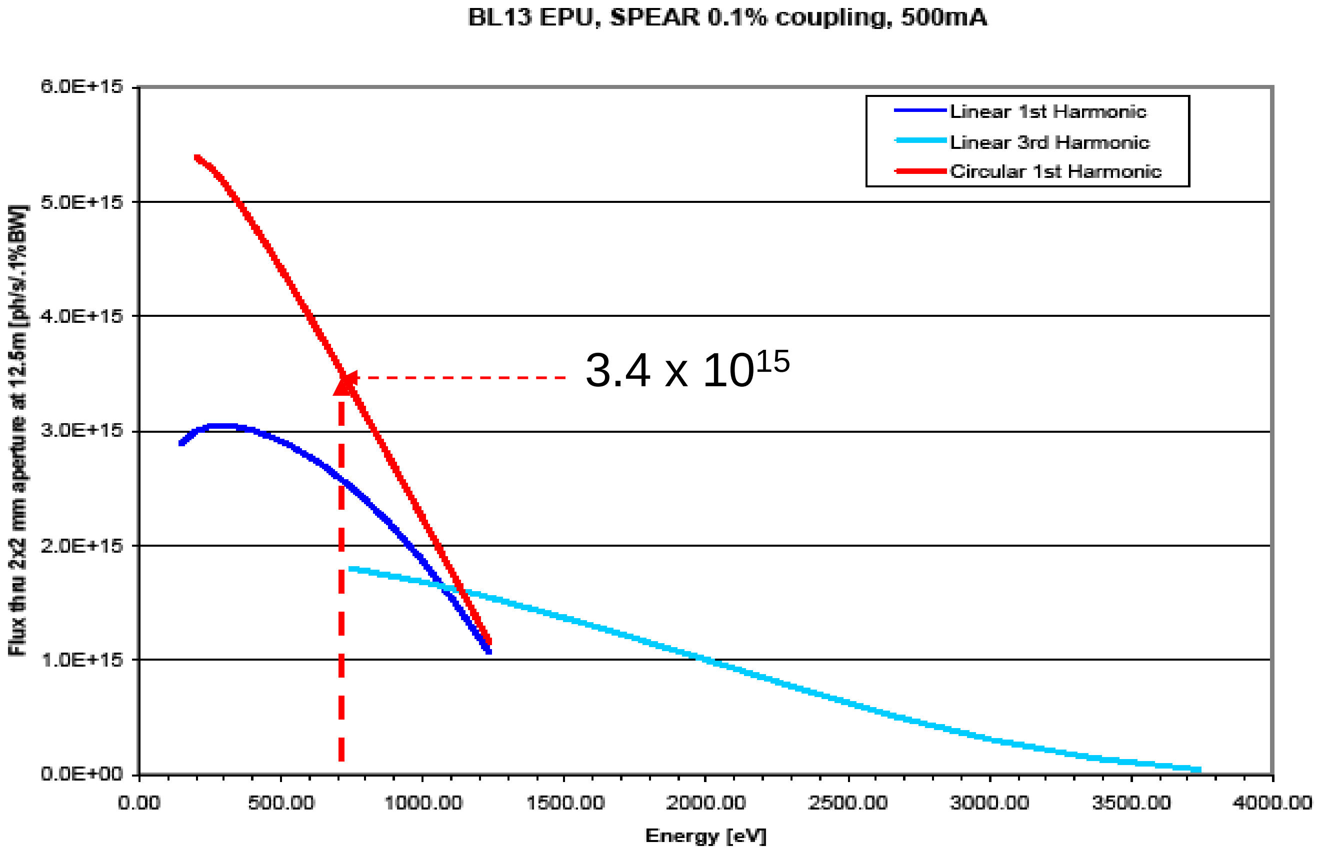
- The required field strength can be sustained by only compensating for RF losses in the cavity

Theory of microwave undulator



- Electrons can interact with both forward and backward flowing RF waves
- Cavity dimensions are usually large to keep losses low and to reduce sensitivity to dimensions

Calculated flux curves for Elliptical Polarized Undulator



3.4×10^{15}

# of periods = 65, λ_u = 5.69cm, K=1.07, Beam current = 500 mA

Requirements of SPEAR ring RF undulator to be designed

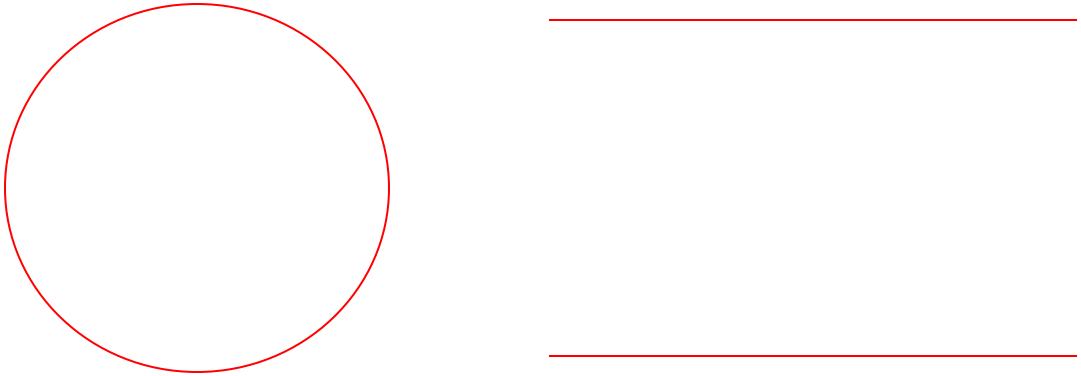
- Electron beam energy: 3 GeV, beam current: 500 mA, aperture: 2 x 2 mm
- Radiation energy: 700-900 eV (circular polarization)
- Fast polarization switch-ability
- Average photon flux and brightness to be within a factor of **10** compared to static magnetic field Elliptical Polarization Undulator

Choice of waveguide and modes

- RF field should be “rotate-able” to control radiation polarization
- We consider only circular symmetric waveguide
- Operating RF mode should have strongest field on waveguide axis along path of electron beam
- Very important to keep wall losses extremely low
- Wall losses determine the cost and feasibility of microwave power source

Waveguide modes considered

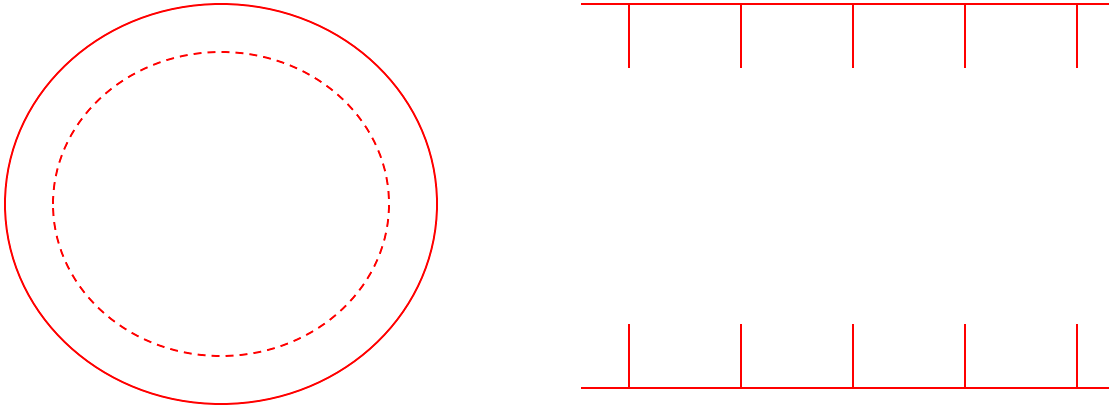
Cylindrical waveguide



TE$_{11}$ mode

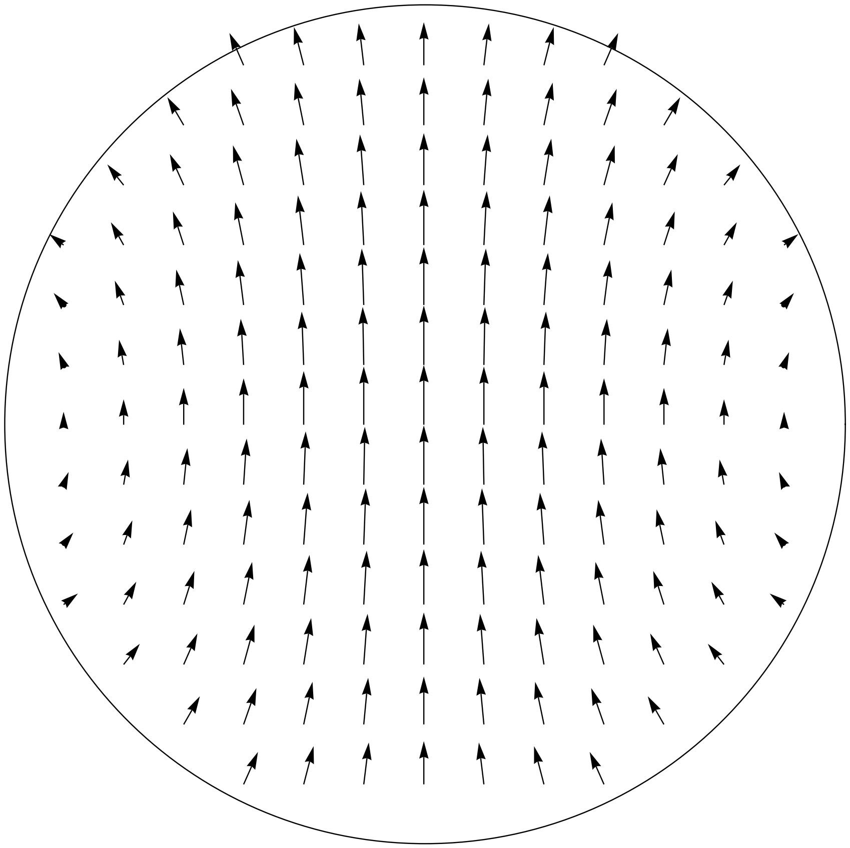
TE$_{12}$ mode

Corrugated waveguide



HE$_{11}$ mode

Circular waveguide mode TE_{11}



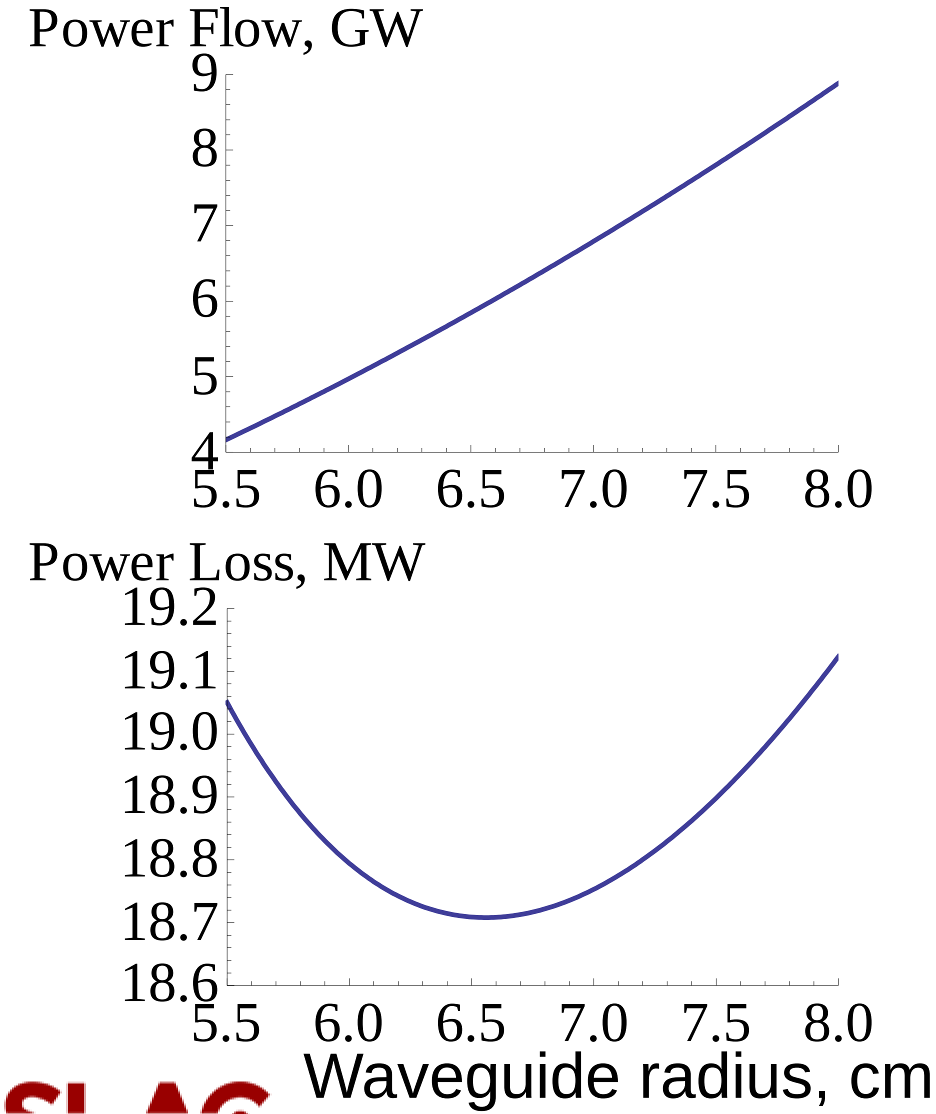
- Fundamental mode, easily excitable
- Has very strong RF field on the axis where the electron bunch travels
- Not the least lossy mode

Electric field over waveguide cross-section

SLAC NATIONAL ACCELERATOR LABORATORY

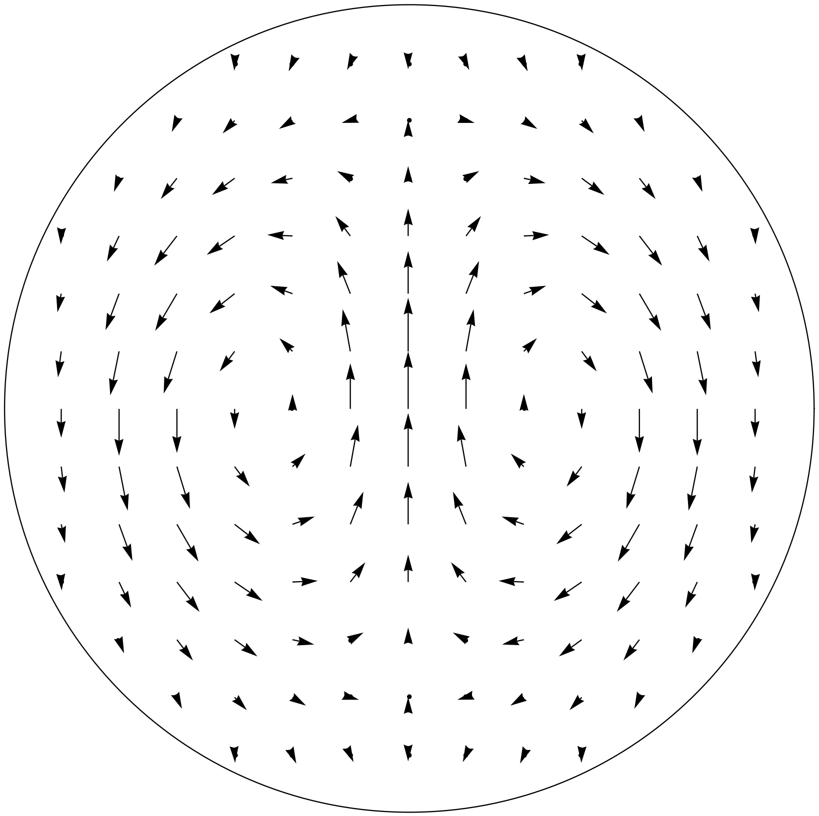
BROOKHAVEN NATIONAL LABORATORY

Power loss in a TE_{11} circular waveguide undulator



- **Flux = 1/5th of Static undulator flux (3.4 x 10^{15} [ph/s/0.1 % BW)**
- **K = 0.71**
- **Circular polarization**
- **Photon energy = 700 eV**
- **L = 3.7 m**
- **λu = 6.1 cm**
- **f = 2.6 GHz**

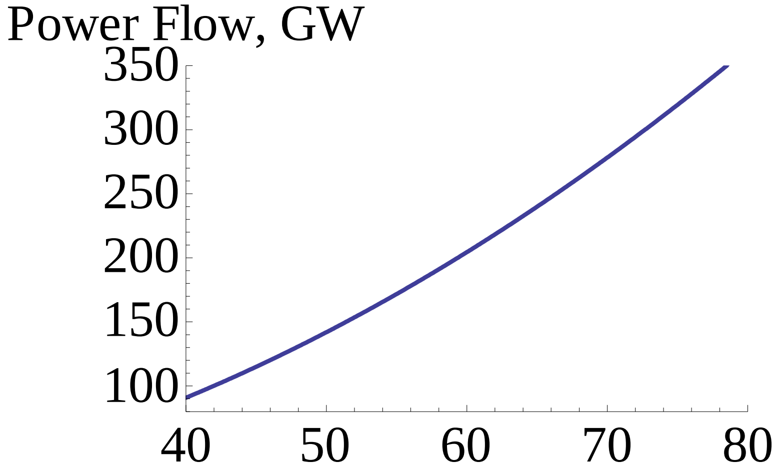
Circular waveguide mode TE_{12}

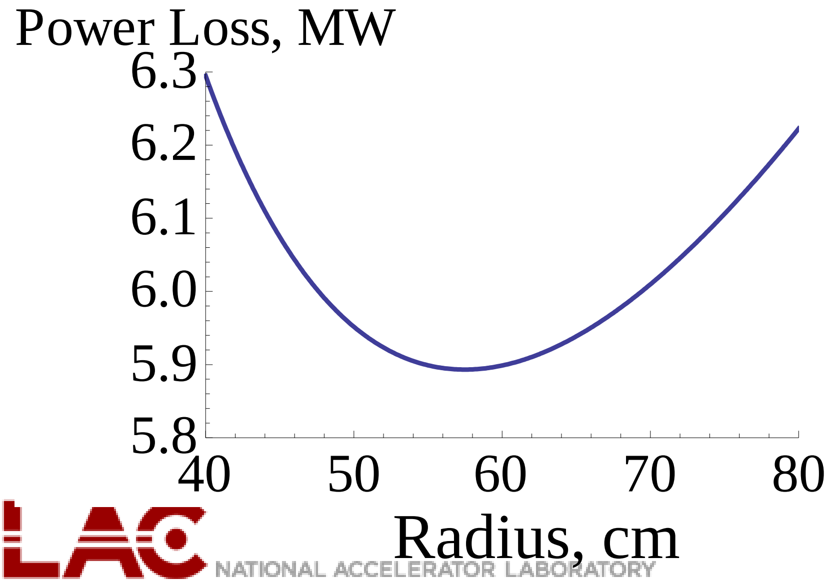


Electric field over waveguide cross-section

- **Has same field structure at the axis as TE_{11} mode**
- **Needs much larger waveguide radius and power**
- **Attenuation is much lower than TE_{11} mode**

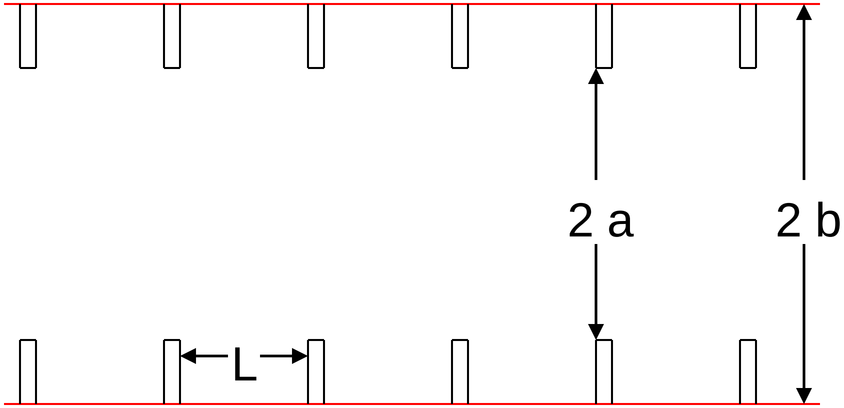
Power loss in a TE_{12} circular waveguide undulator

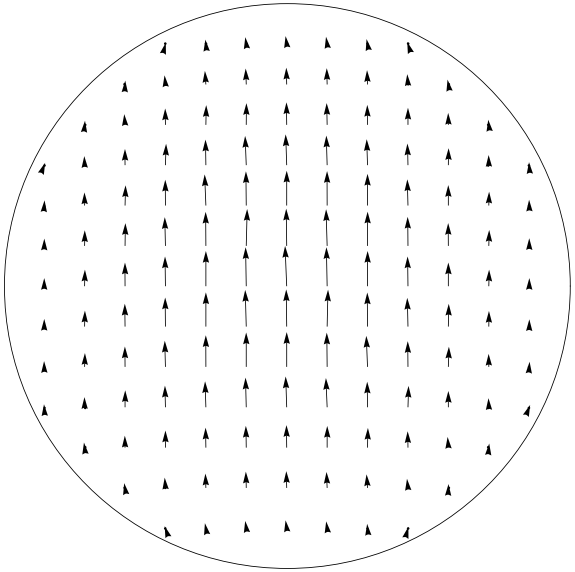




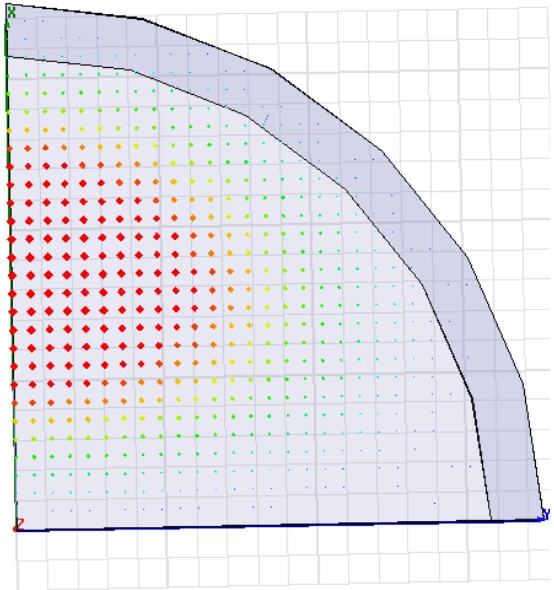
- **Flux = 1/5th of Static undulator flux (3.4 10^{15} [ph/s/0.1 % BW)**
- **K = 0.68**
- **TE12 mode, Circular polarization,**
- **Photon energy = 700 eV**
- **L = 3.7 m**
- **λ_u = 6.35 cm**
- **f = 2.38 GHz**

Corrugated waveguide, hybrid HE_{11} mode

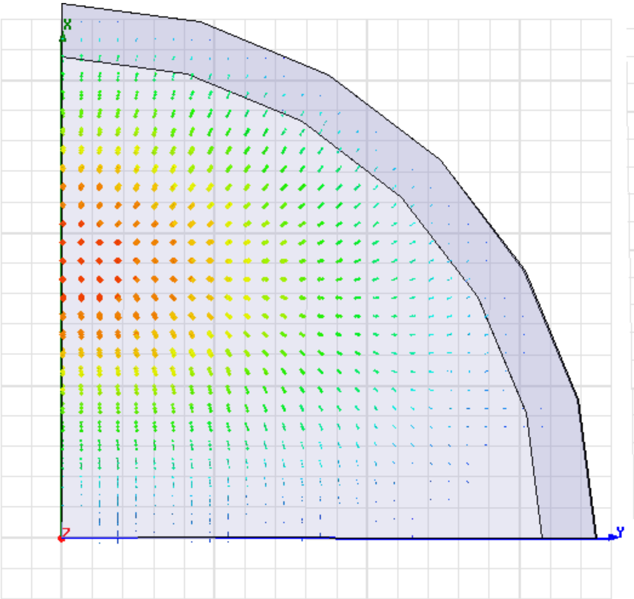




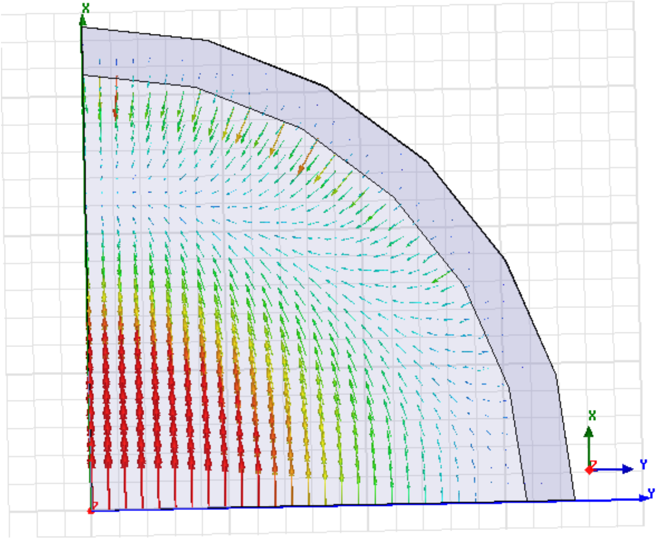
- Combined TE and TM modes lead to hybrid modes
- Under “balanced hybrid” conditions, the field transforms in to low loss linearly polarized wave
- The field is strongly linearly polarized on the axis which is highly desirable for undulator operation
- The field is extremely low near the waveguide walls translating to very low RF losses



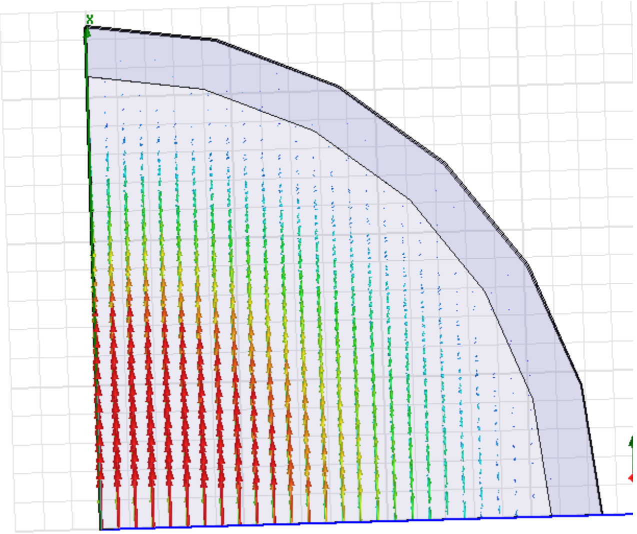
β L = 0.05 rad, Q = 36945



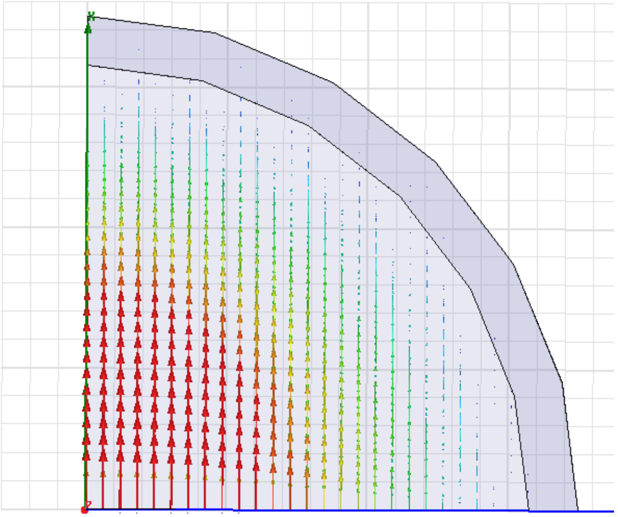
β L = 0.3 rad, Q = 42537



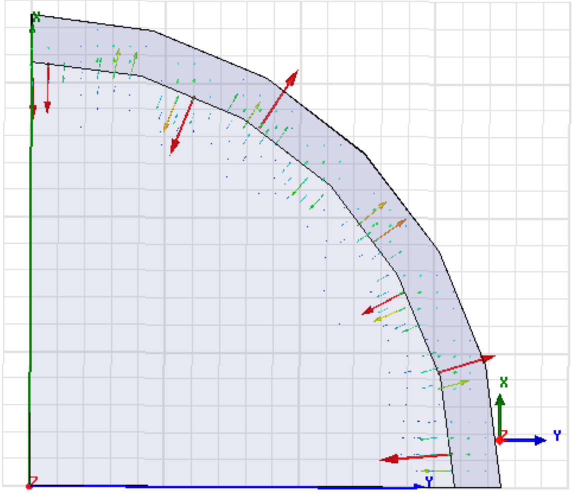
β L = 0. 5 rad, Q = 74322



β L = 1 rad, Q = 1.05 x 10^6



β L = 1.65 rad, Q=5.5 x 10^6



β L = 2 rad, Q=13574

Analysis of corrugated waveguide (neglecting space harmonics)

$r < a$

$$F_z = AJ_m(k_r r)e^{jm\phi}; A_z = AZ_z J_m(k_r r)e^{jm\phi}$$

$$E_z = -j\frac{k_r^2}{k\sqrt{\mu\varepsilon}}A_z; H_z = -j\frac{k_r^2}{k\sqrt{\mu\varepsilon}}F_z$$

$$E_r = -\frac{1}{\varepsilon r}\frac{\partial F_z}{\partial \phi} - \frac{\beta_z}{k\sqrt{\mu\varepsilon}}\frac{\partial A_z}{\partial r}$$

- Both TE$_z$ and TM$_z$ modes are present
- Balanced Hybrid mode is possible only when wave guide impedance $Z_z \approx$ Free space wave impedance

$a < r < b$

$$A_z = jA\frac{\sqrt{\mu\varepsilon}}{k}J_m(k_r a)\frac{H_m^{(2)}(kr)H_m^{(1)}(kb) - H_m^{(1)}(kr)H_m^{(2)}(ka)}{H_m^{(2)}(ka)H_m^{(1)}(kb) - H_m^{(1)}(ka)H_m^{(2)}(kb)}e^{jm\phi}$$

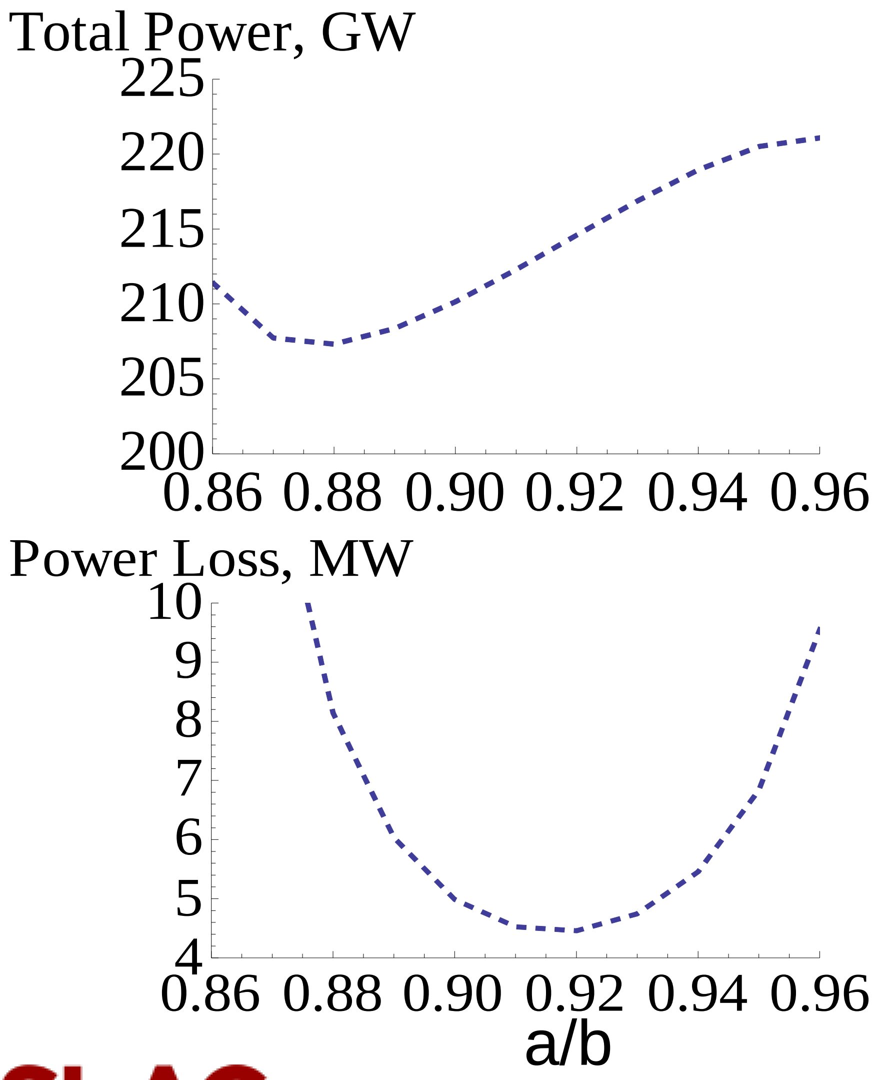
$$E_z = -j\frac{k}{\sqrt{\mu\varepsilon}}A_z; H_r = \frac{1}{\mu r}\frac{\partial A_z}{\partial \phi}; H_\phi = -\frac{1}{\mu}\frac{\partial A_z}{\partial r}$$

- Only TM$_z$ mode present
- Balanced Hybrid condition possible when $(b - a) \approx \lambda/4$

Boundary conditions for corrugated waveguide

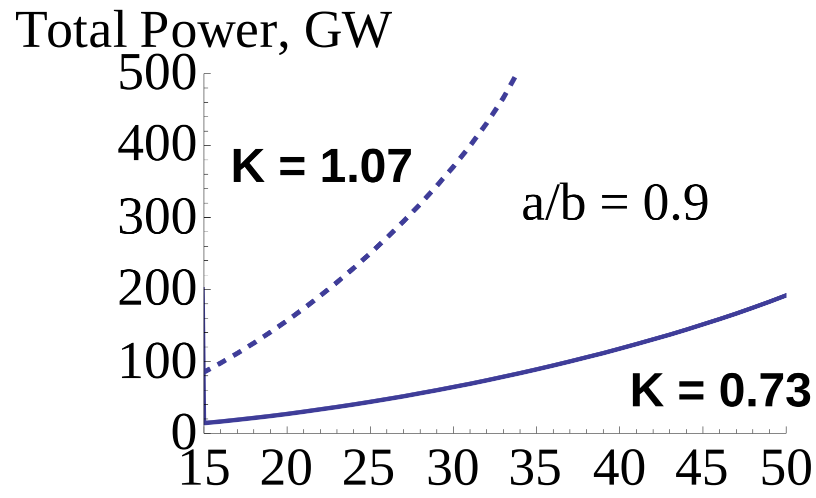
- For sufficiently small slot width, no TE_z mode present inside corrugation
- Then, at $r = a$, $E_\Phi = 0$
- Admittance H_Φ / E_z is continuous at $r = a$
- Equating admittance for the two set of equations at $r = a$ gives the characteristic dispersion equation for the corrugated waveguide

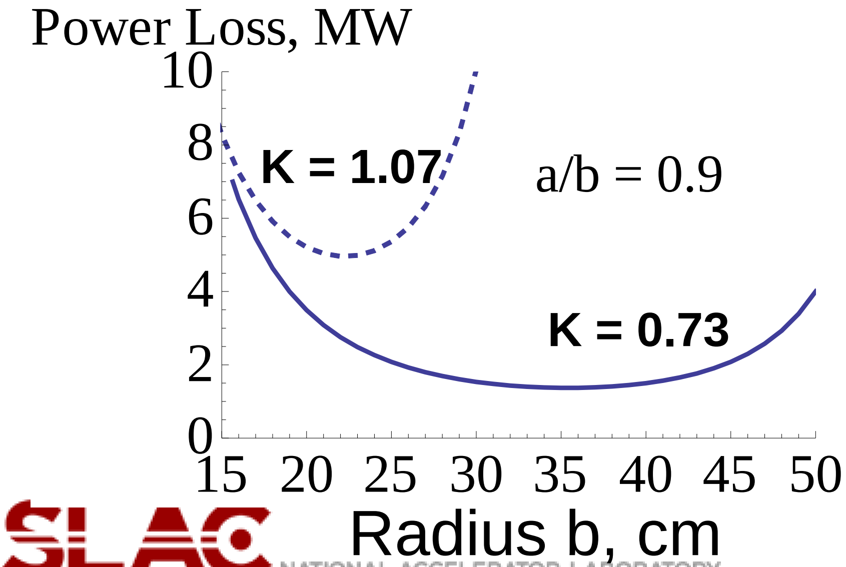
Power loss dependence on corrugation depth



- Flux = Static undulator flux (3.4 10^{15} [ph/s/0.1 % BW)
- **K=1.07**
- **Photon energy = 700eV**
- **L = 3.7 m**
- **b = 23 cm**
- **f = 4.05 GHz**
- **λu = 5.92 cm**

Power loss in a HE$_{11}$ corrugated waveguide undulator





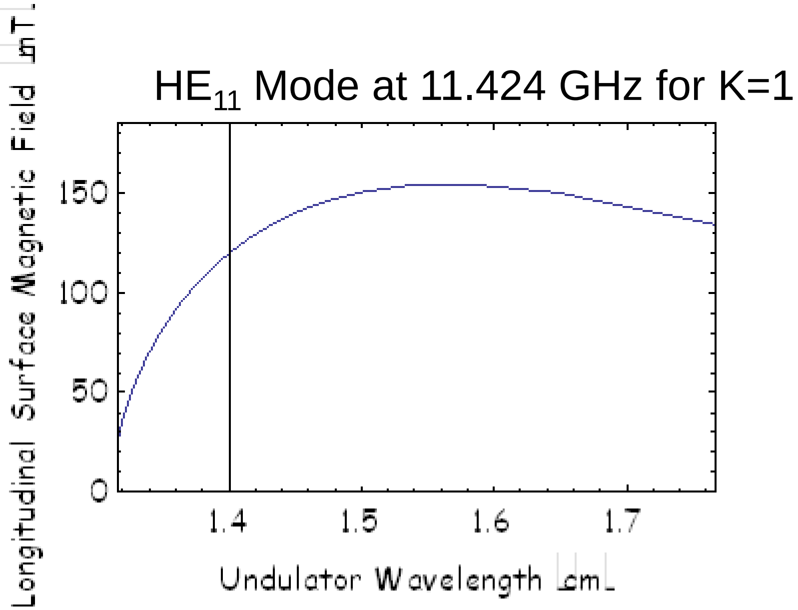
- Flux = Static undulator flux
- **K = 1.07**
- **f = 4.05 GHz**
- **λu = 3.71 cm**

- Flux =1/5th Static undulator flux
- **K = 0.68**
- **f = 2.55 GHz**
- **λu = 5.92 cm**

Superiority of HE_{11} - mode

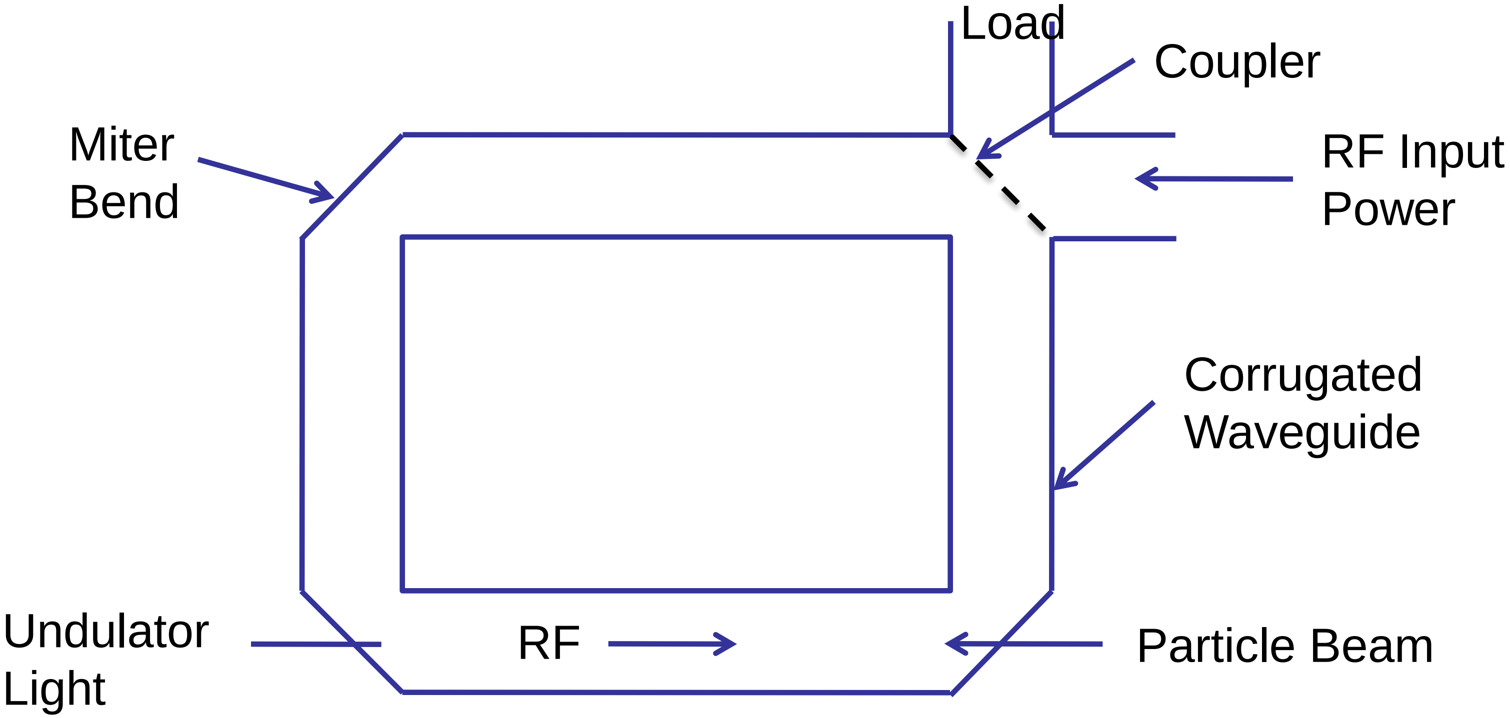
	TE_{11}	TE_{12}	HE_{11}
Undulator parameter K	0.71	0.68	0.68
Power flow (GW)	5.8	180	79
RF power loss (MW/m)	5.1	1.6	0.326
RF frequency (GHz)	2.64	2.38	2.37
Cavity Radius (cm)	6.5	57.7	38

Superconducting RF Undulator



- Surface magnetic field is less than the quenching field of niobium
- Allows application of RF undulator to storage ring applications, where CW or quasi-CW operation are required
- RF power needed is only few hundred watts to kilo watts – sources readily available

Resonant Ring Configuration



- A closed ring with length nλg
- Tune by adjusting ring length
- Considerable development for relevant components (miter bend, couplers) has been done (ITER transmission lines)

Conclusions and future work

- Corrugated waveguide is an attractive option for a microwave undulator due to its low losses
- Successful development will enable design of undulators with capabilities not possible with current static undulators
- Could lead to a new class of FEL and storage ring undulators