A Scaling Law for Planar and Helical Superconducting Undulators

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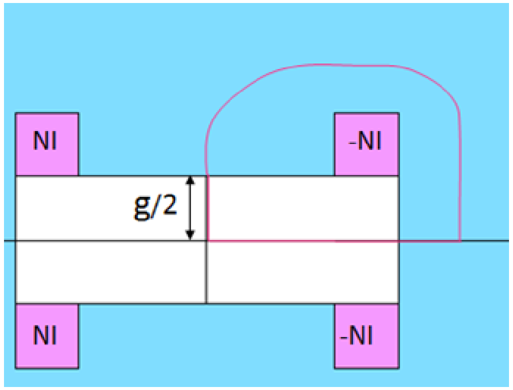
Introduction

Iron dominated magnet [1]

$$NI_{pole} = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} \rightarrow B(g)_{Dipole} \approx \frac{\mu_0 NI_{pole}}{g/2}$$



$$B(r_0)_{Quad} \approx \frac{2\mu_0 NI_{pole}}{r_0}$$

$$B(r_0)_{Sext} \approx \frac{3\mu_0 NI_{pole}}{r_0}$$

B_0 = const jl

Electromagnetic undulator [2]

$$B_n(\mu \approx \infty) = \frac{32\mu_0 NI}{\pi\lambda n}\frac{\sin(n\pi/4)}{\sinh(nkg/2)} \quad (k = \frac{2\pi}{\lambda})$$

$$B_y = \frac{32\mu_0 NI}{\sqrt{2}\pi\lambda}\left[\frac{\cosh(ky)}{\sinh(kg/2)} - \frac{\cosh(3ky)}{3\sinh(3kg/2)}\right]$$

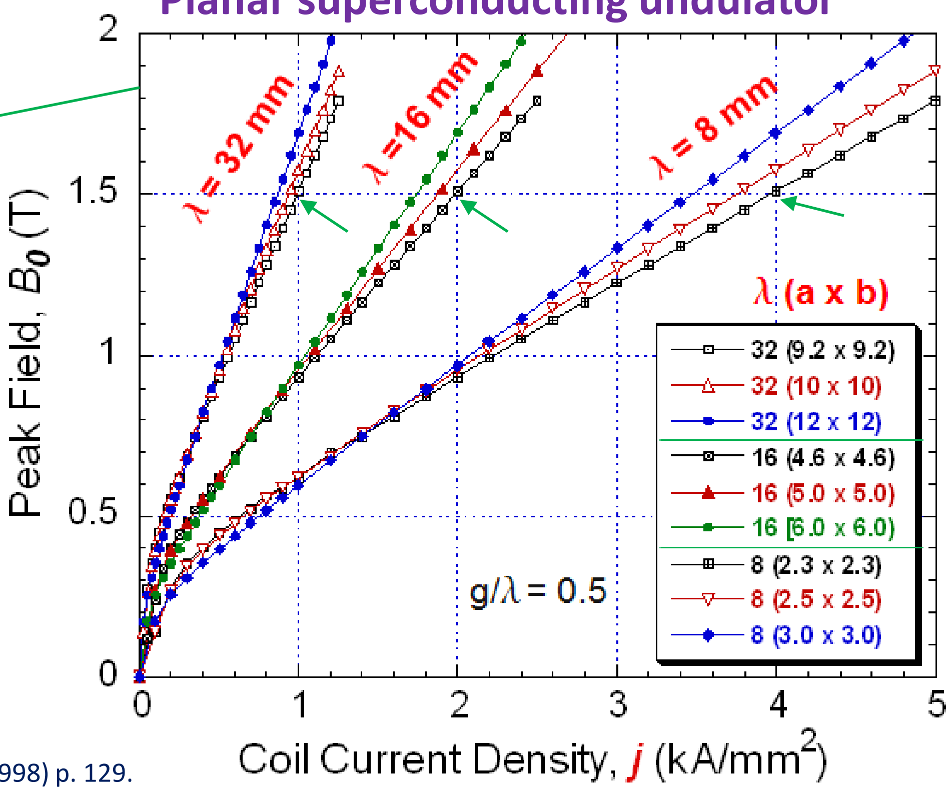
Permanent magnet undulator [3]

$$B_0 = 2B_r \frac{\sin(\pi/m)}{(\pi/m)}(1-e^{-kh})e^{-kg/2}$$

$$B_0 = a \cdot \exp\left[-b\frac{g}{\lambda} + c\left(\frac{g}{\lambda}\right)^2\right], \ (0.07 < \frac{g}{\lambda} < 0.7)$$

SmCo5: Br = 0.9 T, a = 3.33, b = 5.47, c = 1.8
NdFeB: Br = 1.1 T, a = 3.44, b = 5.08, c = 1.54

Planar superconducting undulator



[1] G.E. Fisher, AIP Conf. Proc.No. 153, AIP, New York (1987) p.1122.
[2] R.P. Walker, Nucl. Instrum. Methods, A237,366 (1985); CERN 98-04 (1998) p. 129.
[3] K. Halbach, Nucl. Instrum. Methods, 187 (1981) 109; J. Phys. C1 44, 211 (1983).

Outline

- Introduction
- Analytical expression of planar (electromagnetic) undulator
- $j\lambda$ scaling and Ampere's law
- Data on pole-gap dependence and coil dimensions to support the $j\lambda$ scaling
- Characteristics, $j\lambda$ scaling, and analytical expression of bifilar helical undulator
- An example of the $j\lambda$ scaling for a superconducting undulator (SCU): Why it is much more difficult with "short-period" SCUs
- Summary

Planar undulator

Analytical expressions [1]

$$j_x(z) = \sum_{n=1,3,\cdots}^{\infty} \frac{4j}{n\pi} \sin(nk\frac{a}{2})\cos(nkz) \quad (k = \frac{2\pi}{\lambda})$$

per unit height of the coils

$$B_y(y,z) = \sum_{n=1,3,\cdots} B_n \cosh(nky)\sin(nkz)$$

$$B_z(y,z) = \sum_{n=1,3,\cdots} B_n \sinh(nky)\cos(nkz)$$

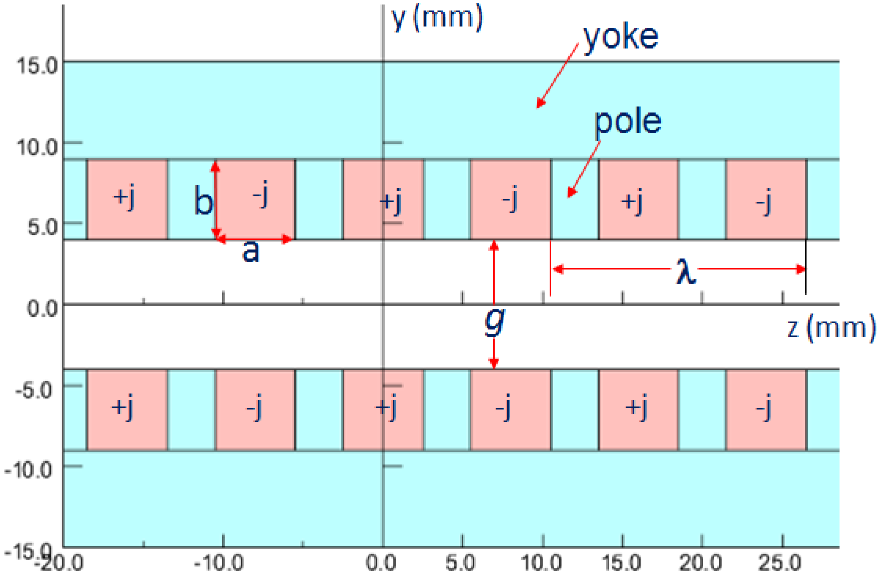
$$B_n = \frac{2\mu_o j\lambda}{(n\pi)^2}\sin(nk\frac{a}{2})e^{-nkg/2}\left(1-e^{-nkb}\right)$$

$$B_y(0,z) = B_0 \sin(kz)$$

$$B_z(0,z) = B_0 \cos(kz)$$

$$B_0 = \frac{2\mu_o j\lambda}{\pi^2}\sin(k\frac{a}{2})e^{-kg/2}\left(1-e^{-kb}\right)$$

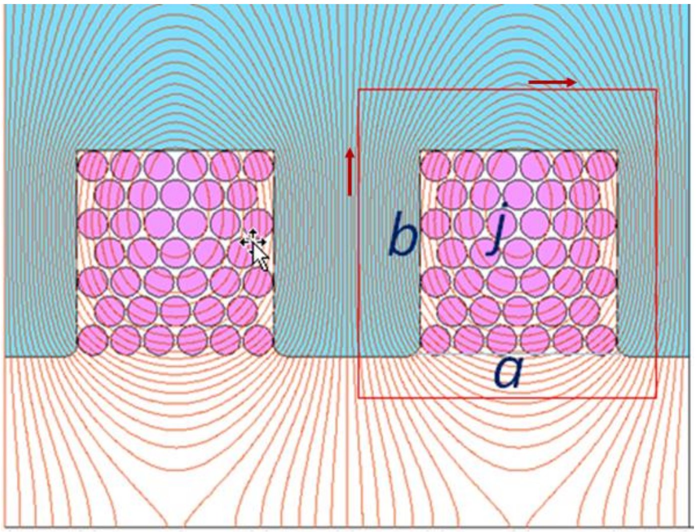
Schematic 2D cross section in (x = 0)



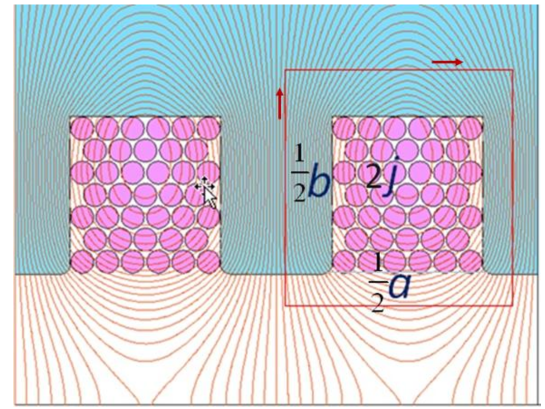
- When SCU dimensions are scaled, with l as the reference, B_0 remains unchanged for a constant *j*l.
- The on-axis peak field depends approximately on exp(-*kg*/2).
- The derived equation B_n predicted the third harmonic of the on-axis field correctly.
- Surprisingly, the *j*l scale holds for undulators with nonlinear magnetic materials.

[1] S.H. Kim, Nucl. Instrum. Methods, A 546 , 604 (2005).

Does Ampere's law explain the *jl* scaling ?



$$\lambda \to \frac{1}{2}\lambda$$



NI

$$abj = \int_{air} \frac{\vec{B}}{\mu_0} \cdot d\vec{l} + \int_{iron} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l}$$

$$\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right)(2j) = \int_{air} \frac{\vec{B}}{\mu_0} \cdot d\vec{l} + \int_{iron} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l}$$

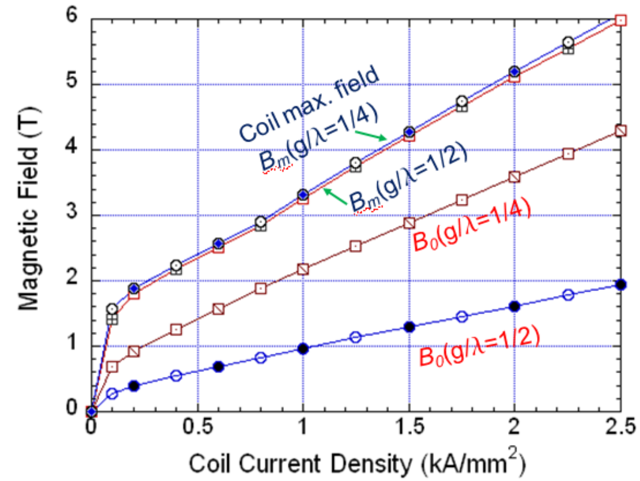
$\frac{1}{2}NI$

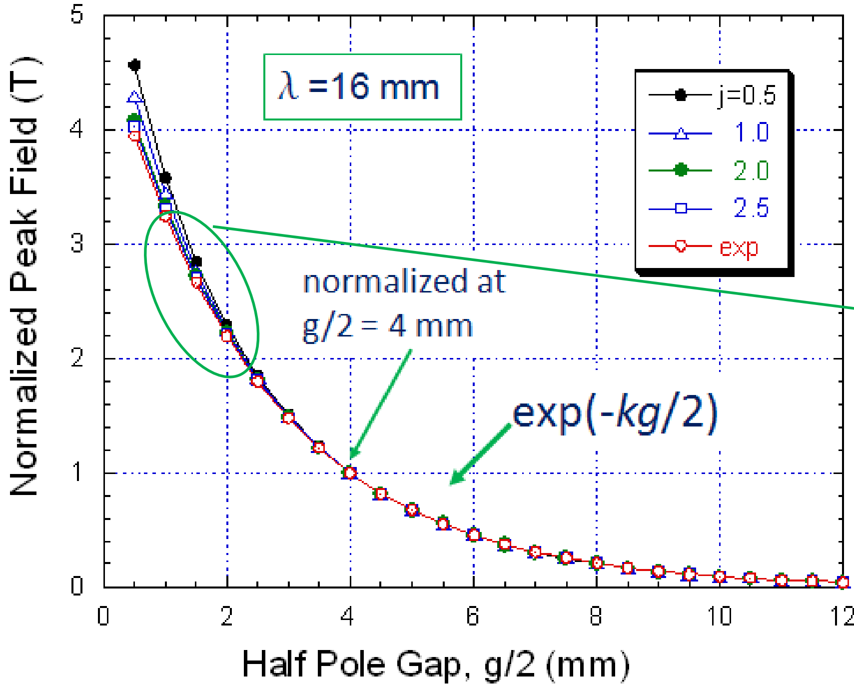
$d\vec{l} = \frac{1}{2}\, d\vec{l}$

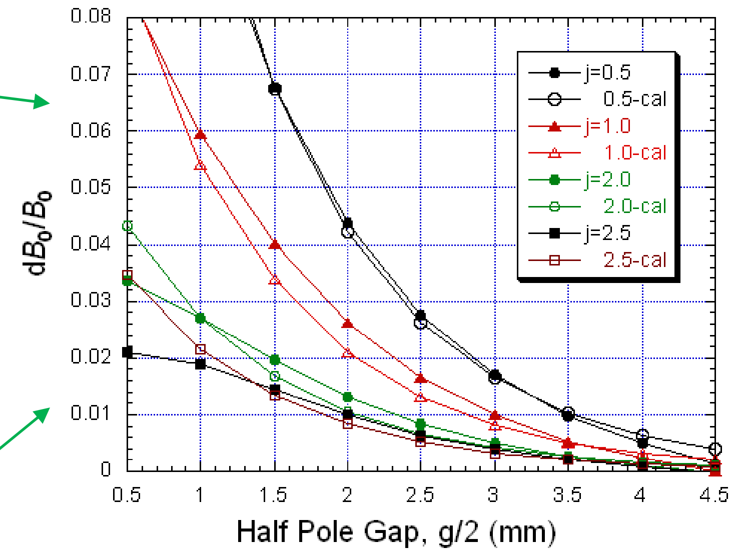
- For a constant *jl*, if the flux density distributions remain unchanged in the above two geometries (which include the nonlinear magnetic poles and flux-return yokes), then the *jl* scaling may be understandable with Ampere's law.
- Numerical analysis showed that the flux density, as well as the permeability, distributions of the two were identical [1].

[1] Opera, Vector Fields Software, Cobham Technical Services, Aurora, IL 60505, USA

Peak field B_0 dependence on g/λ for nonlinear cases



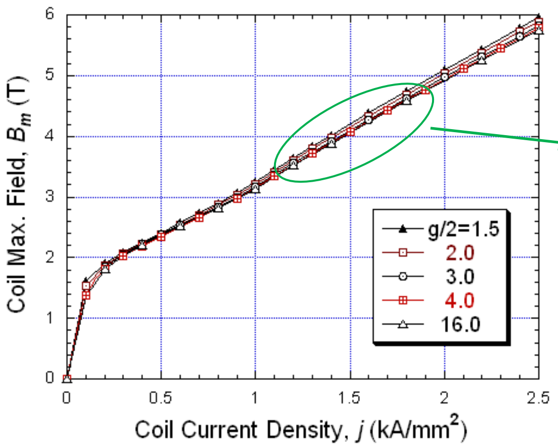


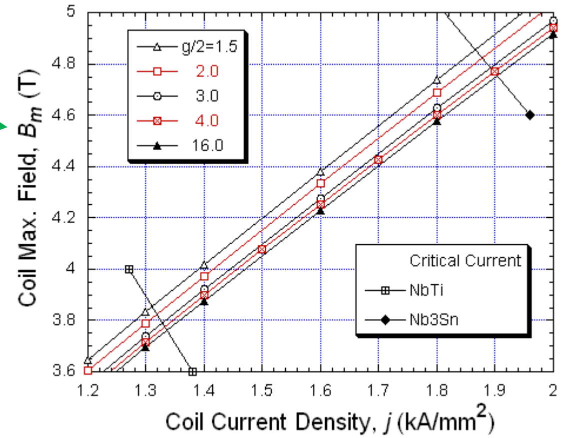


$$B_0(j,\frac{g}{\lambda}) = B_0(j,0.5)\exp[-\pi(\frac{g}{\lambda}-0.5)]\times\{1+\delta j_n^{-1}\exp[-2.4\pi(\frac{g}{\lambda}-0.5)]\}$$ Eq. (1) $(\delta = 0.0016)$

- For $g/\lambda > 0.4$, B_0 is proportional to exp(-*kg*/2) within 1%.
- The above equation, which has a correction term, holds for $g/\lambda > 0.06$ within 1%.

Coil max. field B_m dependence on g/λ

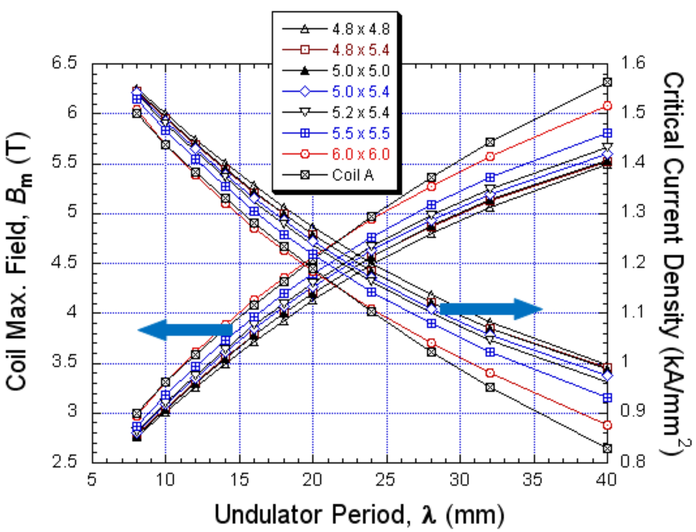


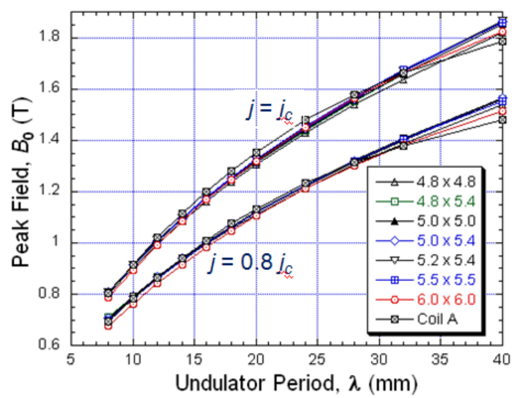


- When the pole gap is reduced, the coil maximum field increases, which, in turn, reduces the SC critical current.
- Therefore, the correction term for g/λ > 0.4 in Eq. (1) may be neglected.

Optimized coil dimensions?

***g*/l= 0.5**
NbTi SC





- Bigger coil dimensions make higher coil maximum fields, which reduce the SC critical current densities.
- Therefore, the spread of the peak fields reduced, and the dependence of the achievable peak fields on coil dimensions is relatively small.

Helical undulator

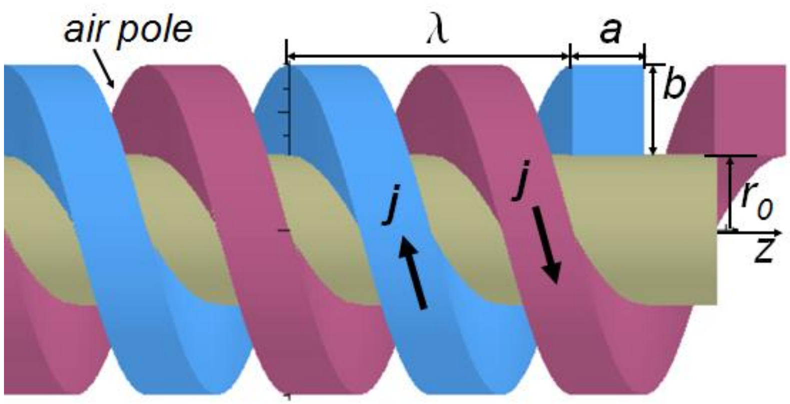
Helical Solenoid [1]

$$B_{tr} = \frac{\mu_0 I}{\lambda}\{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

Helical undulator [2]

$$B_0 = \frac{2\mu_0 I}{\lambda}\{kr_0 K_0(kr_0) + K_1(kr_0)\}$$

($k = 2\pi/\lambda$) (Current I in a filament wire)



For I_1 and I_2 in opposite directions in each helix:

$$B_0 = \frac{\mu_0(I_1 + I_2)}{\lambda}\left[kr_0 K_0(kr_0) + K_1(kr_0)\right] \qquad B_z = \frac{\mu_0(I_1 - I_2)}{\lambda}$$

Helical Undulators with coil dimensions (a, b) [3]

$$\mathbf{B}_< = \sum_{n=1,3,5..}^{\infty} B_0^n \cdot [\hat{r}\{I_{n-1}(nkr) + I_{n+1}(nkr)\}\sin n(kz-\phi) + \hat{\phi}(-2/kr)I_n(nkr)\cos n(kz-\phi) + \hat{z}2I_n(nkr)\cos n(kz-\phi)]$$

$$B_0^n = \frac{2\mu_0 j}{\pi}\sin\left(\frac{nka}{2}\right)\left[kr_0 K_{n-1}(nkr_0) + K_n(nkr_0)\right] \qquad I_m(x) = \sum_{\ell=0}^{\infty}\frac{(x/2)^{m+2\ell}}{\ell!(m+\ell)!}, \quad (m \ge 1)$$

$$\mathbf{B}(kz-\phi) = B_0\left\{\hat{r}\sin(kz-\phi) - \hat{\phi}\cos(kz-\phi)\right\} \qquad B_0 = \frac{2\mu_0 j\lambda}{\pi}\sin(k\frac{a}{2})\int_{r_0}^{r_0+b}\{krK_0(kr) + K_1(kr)\}\frac{dr}{\lambda}.$$

- On-axis field has no higher harmonic fields, while the off-axis field components do.
- For a constant $j\lambda$, B_0 remains unchanged when length dimensions are scaled.
- How about with steel poles?

[1] *W.R. Smythe, Static and Dynamic Electricity* (McGraw-Hill, New York, 1939), p. 272.
[2] B.M. Kincaid, J. Appl. Phys. 48, 2684 (1977); J.P. Blewett and R. Chasman, *abid.* 48, 2692 (1977).
[3] S.H. Kim, Nucl. Instrum. Methods, A 584, 266 (2008).

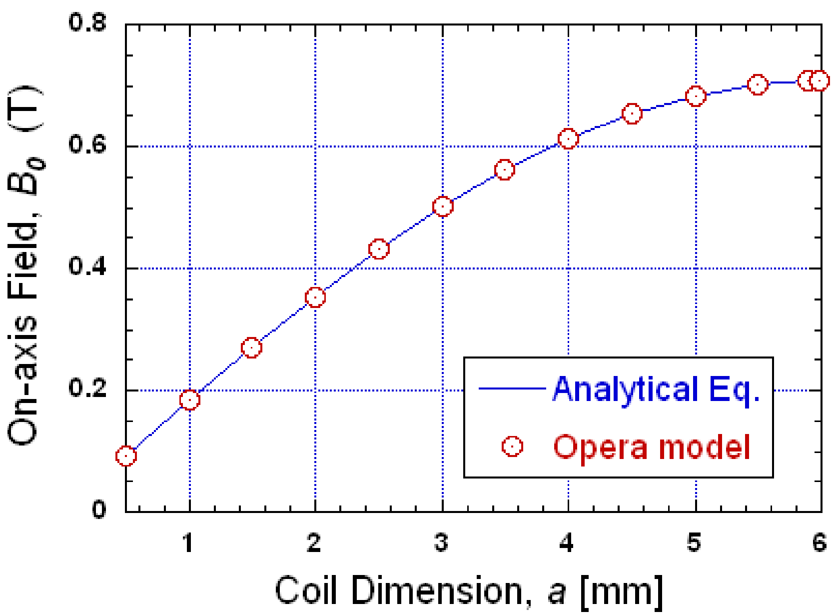
Helical undulator:
Model calculation to verify the analytical results

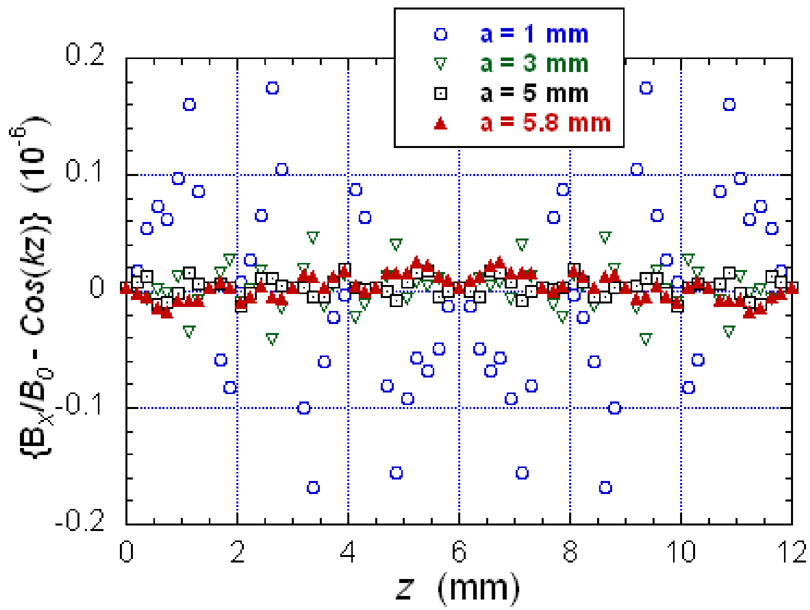
period = 12 mm, coil r_0 = 3.15 mm
coil dimension b = 3.84 mm
j(coil) = 1kA/mm^2
$(B_{OP} - B_{Eq})/B_{Eq} < 0.1\%$

$$[K] \approx \exp(-0.95kr)$$

$$B_0 = 0.8 j \lambda \sin(\frac{ka}{2}) \int_{r_0}^{r_0+b} [krK_0(kr) + K_1(kr)] \frac{dr}{\lambda}$$

$B_0[T]$, $j[kA/mm^2]$, $\lambda[mm]$

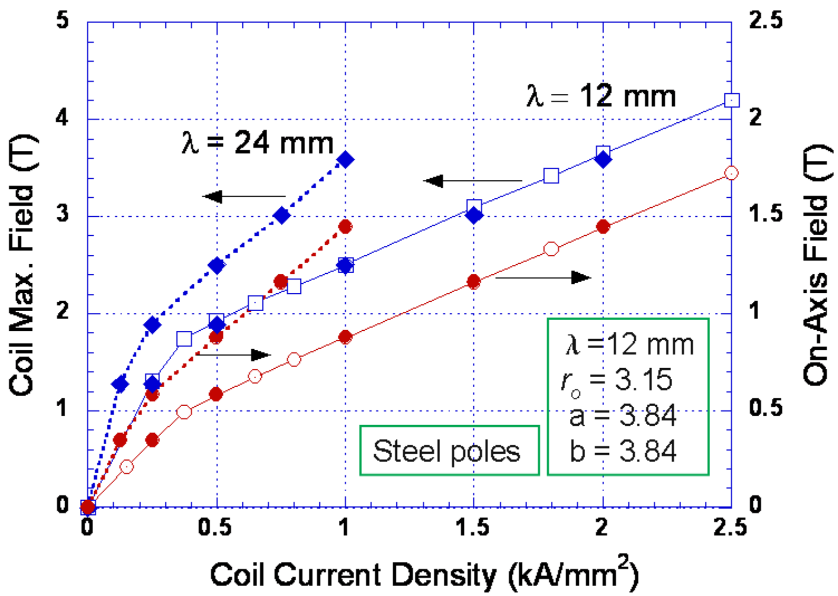


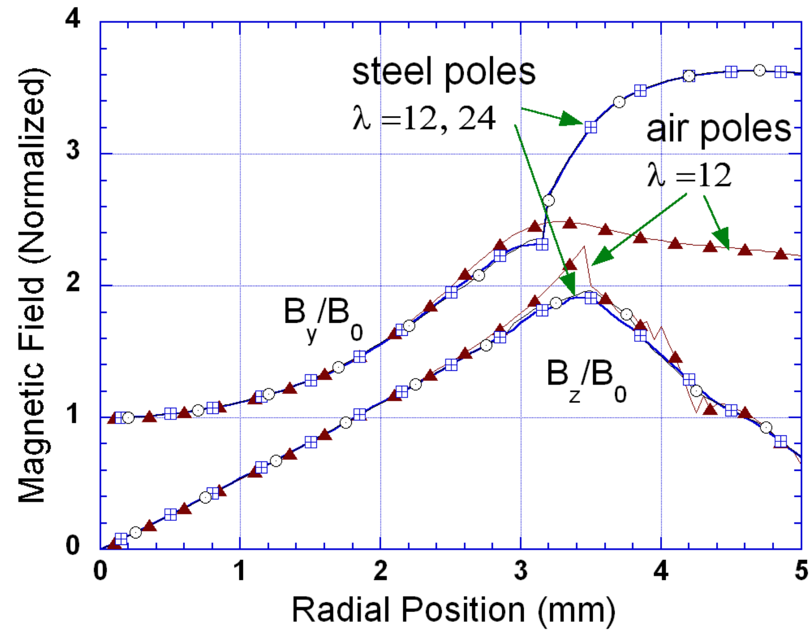


- Analytical equation agrees with model coil calculations within 0.1%.
- Higher harmonic coefficients of the on-axis field are less than 10^{-6}.

Helical undulator: Scaling law for steel poles

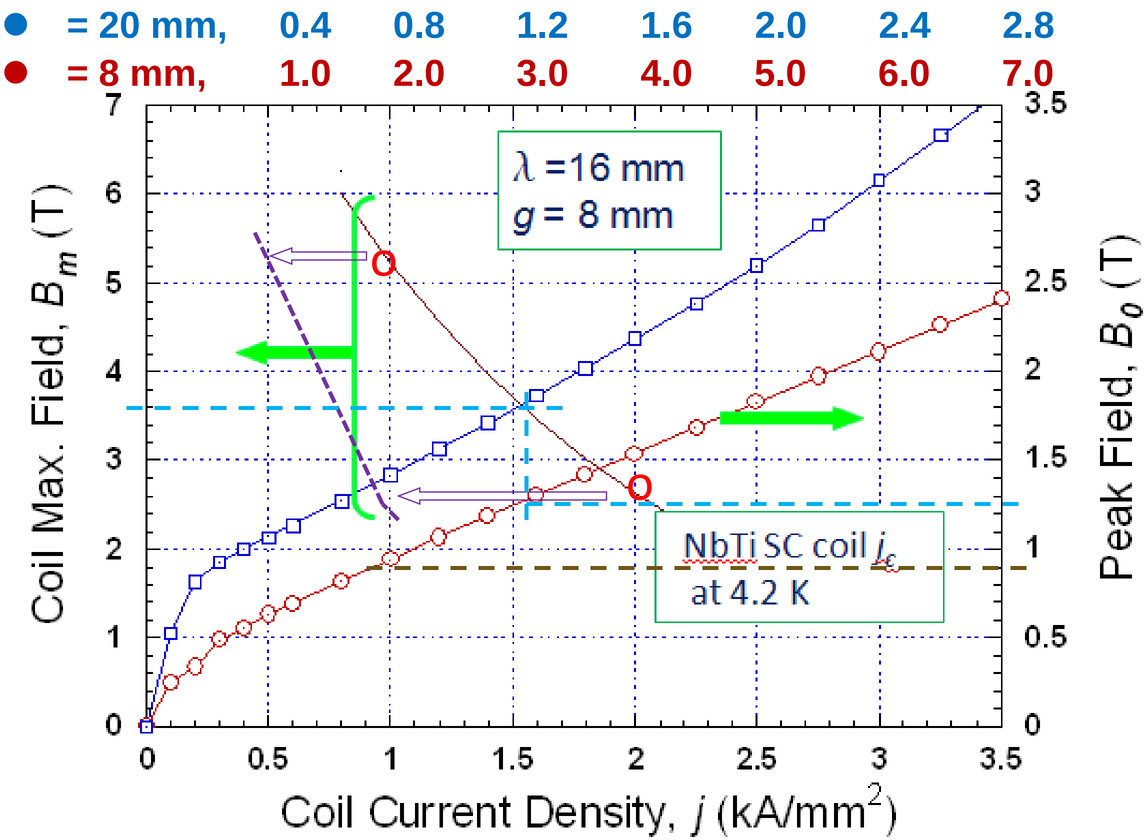
λ = 12, j = 1.0 kA/mm^2, B = 0.877286 T, steel poles
λ = 24, j = 0.5 kA/mm^2, B = 0.876782 T, steel poles, ΔB = 5x10^{-4}
λ = 12, j = 1.0 kA/mm^2, B = 0.577 T, air poles



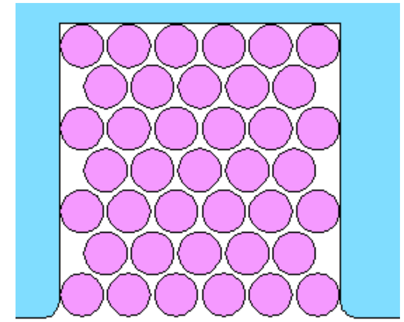


- Off-axis field components follow the $j\lambda$ scaling within 0.1%.
- On-axis field and coil maximum field for the two periods follow the $j\lambda$ scaling.
- When the off-axis field components are normalized to the on-axis fields for the steel poles and air poles, the field components follow the $j\lambda$ scaling for r < 3 mm.

How to use the j● scaling law and why it is so difficult with "short-period" planar and helical SCUs



- Don't be discouraged. If you increase the period from 16 to only 20 mm, there will be big rewards!



an example:

Scale from ●=16 to ●= 8 mm

- B_m = 3.6 T, j_c = 1.55 (@ $\lambda = 16$)
- B_0 = 1.25, K = 1.868 (@ $\lambda = 16$)
- $j\lambda \rightarrow (2j)(\lambda/2)$: same B_m, B_0 data
- $K \rightarrow 0.5K$ (1.868→0.934)
- $g \rightarrow 0.5g$ (8 mm→4 mm)
- j_c(SC)-curve → not 2j_c(SC)-curve
- j_c = 1.55 →1.8, B_0 = 1.25 →0.9 T
- K = 0.934 →0.672
- $B_0(g_1) = (B_0)\exp\{-\pi(g_1-4)/8\}$
- B_0(6 mm) = 0.41 T
- New K = 0.305
- All at NbTi j_c at 4.2 K.

Summary

- The derived analytical expressions of B_0 for planar and helical undulators show that, when length dimensions are scaled according to a period ratio, the field remain unchanged for *a* constant $j_0\lambda_0 = j_n\lambda_n$.
- The $j\lambda$ scaling law extends to the distributions of the flux density and permeability for the whole region of SCUs with nonlinear poles and yokes.
- With one data set for B_0 and B_m, B_0 for different periods may be calculated.
  - B_0 dependence on coil dimension is insignificant.
- For g/λ > 0.15, the peak field varies as $$B_0\left(\frac{g}{\lambda}\right) = B_0\left(\frac{g_1}{\lambda}\right)\exp\left[-\pi\left(\frac{g}{\lambda} - \frac{g_1}{\lambda}\right)\right]$$
- As an example for using the $j\lambda$ scaling law, it was shown why achieving a "required" B_0 for a "shorter period" is an issue.