Analysis of Pulsed Wire Magnetic Field Measurements for Undulators

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09-20-10

Outline

- Motivation
- Introduction to the pulsed wire method
- Analytical and numerical analyses of the pulsed wire method
- Experimental setup and measurement examples
  - Have not had an undulator available for measurements
  - Prototype measurements on permanent dipole magnets
  - Magnetic alignment of pulsed solenoids
- Conclusions

Magnetic Measurements of Undulators

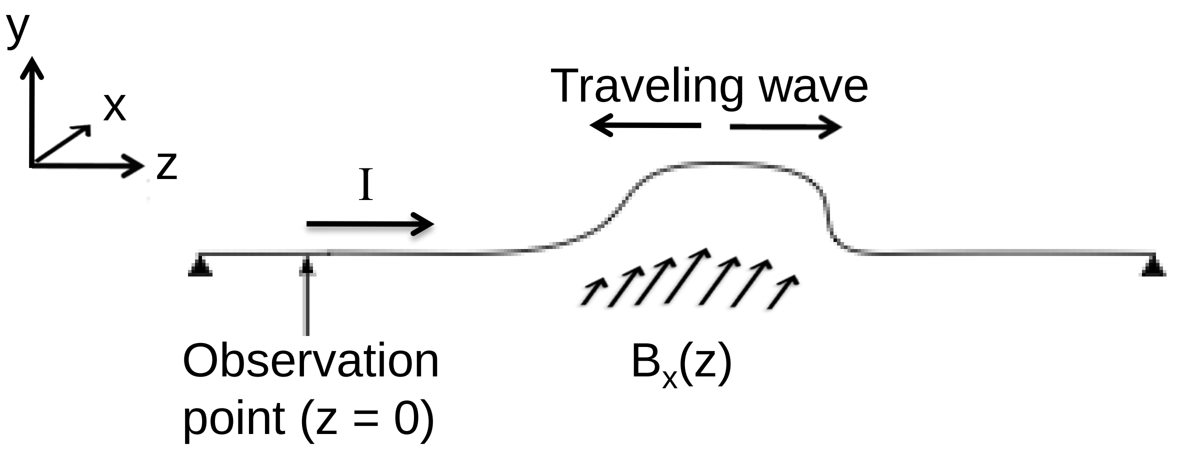
- There is a current R&D effort to develop superconducting undulators for next generation light sources including short period / small gap devices
- Magnetic measurements on undulators can be achieved today with hall probe measurement benches
  - Measurement bench provides accurate field measurements and precise spatial resolution
  - Need access to the undulator gap through the side
- There is a need to develop magnetic measurement systems to characterize undulators with small gaps and/or operating at cryogenic temperatures where the limited access makes magnetic measurements difficult
- Pulsed wire is an attractive candidate for measurements where there are space limitations and limited access
- The accuracy of the method must be improved to be a competitive magnet measurement choice

Pulsed Wire Method

- Method developed by R.W. Warren for the measurement and correction of Wigglers at Los Alamos National Laboratory (1987)
- Method was used to measure the first and second integrals of the magnetic field of the Wigglers
- Warren identified and described some of the limitations of the method in a publication (1988)
- Pulsed wire method was used for the alignment of solenoids for the Integrate Test Stand in the Dual-Axis Radiographic Hydrodynamic Test (DARHT) facility (J.G. Melton et. al., 1993)
- Several improvements to the pulsed wire method for the measurement of undulators (Varfolomeev et. al., 1994 – 1997)
- A. Temnykh modified pulsed wire technique by using an ac pulse at the wire resonant frequencies (Vibrating wire technique, 1997)

Pulsed Wire Description

- Tensioned wire between two points
- Part of the wire is in an external magnetic field
- Current is applied to the wire
- The wire is subjected to the Lorentz force
- A traveling wave moves along the wire
- The displacement at a given point is measured
- The displacement of the wire as a function of time is related to the spatial dependence of the magnetic field



Mathematical Description

- Idealized wire with no flexural rigidity
- Assumed constant tension and density
- Waves propagate without distortion

$$\rho \frac{\partial^2 y}{\partial t^2} - T \frac{\partial^2 y}{\partial z^2} = -\rho g + B_x I$$

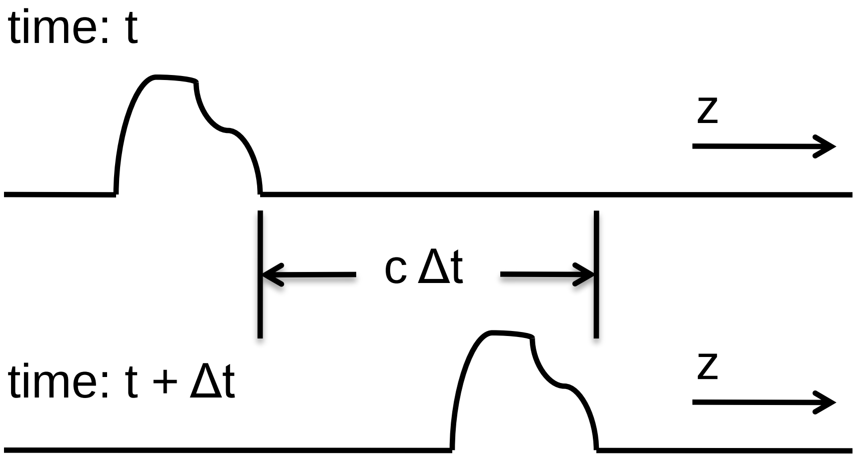
$$c = \sqrt{\frac{T}{\rho}}$$

ρ: wire mass per unit length

T: wire tension

c: wave speed

Wave Propagation



Analytical Solution

- Solution for the wire motion at a given location as a function of time
- A square current pulse with pulse time length δt is assumed
- Result is obtained using the Green's function solution

$\hat{y}(t)$: wire position at z = 0 as a function of time

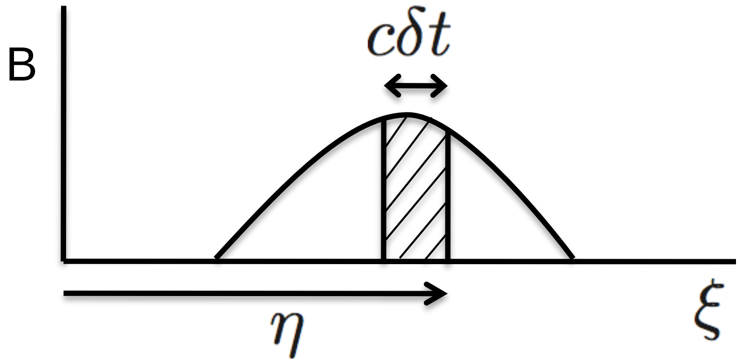
General solution:

$$\hat{y}(t) = \frac{I}{2T} \int_{z=0}^{z=ct} \int_{z=\eta-c\delta t}^{z=\eta} B_x(\xi) d\xi d\eta$$

$$G(\eta) = \int_{z=\eta-c\delta t}^{z=\eta} B_x(\xi) d\xi$$

DC current:

$$\hat{y}(t) = \frac{I}{2T} \int_{z=0}^{z=ct} \int_{z=0}^{z=\eta} B_x(\xi) d\xi d\eta$$



δt → 0:

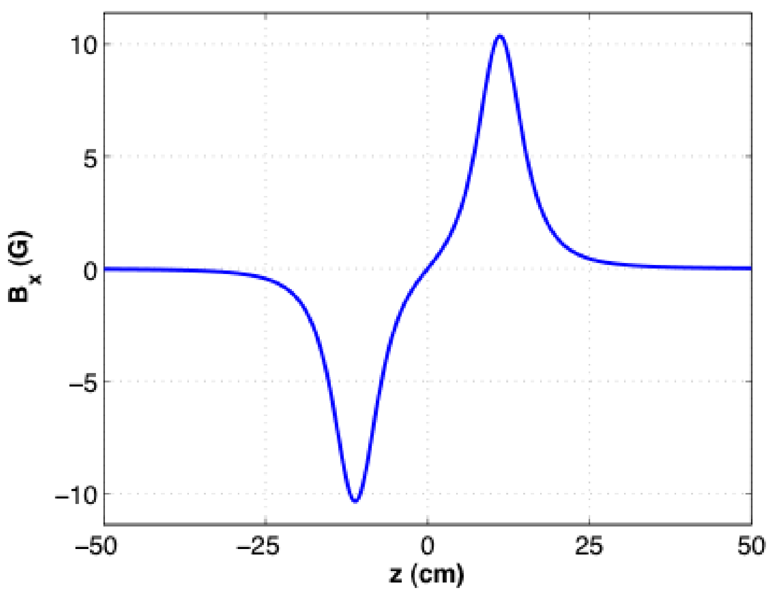
$$\hat{y}(t) = \frac{I c\delta t}{2T} \int_{z=0}^{z=ct} B_x(\xi) d\xi$$

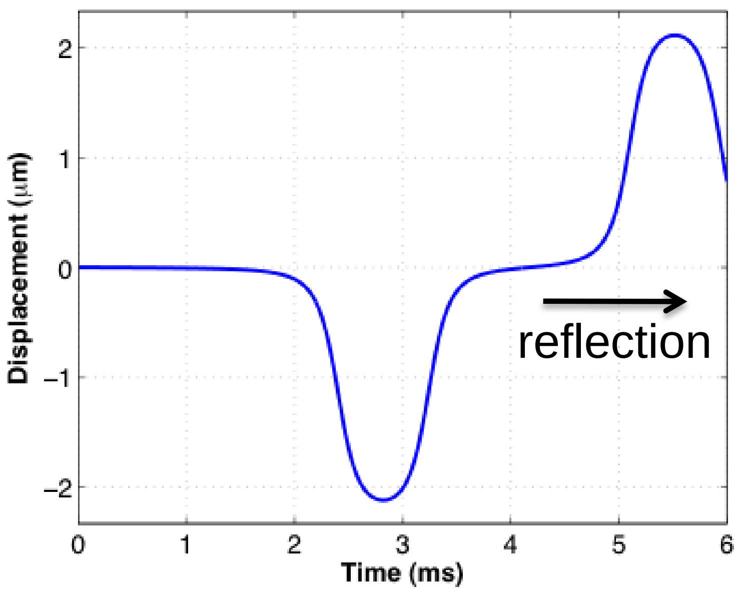
Numerical Solution

- Numerical solution of the partial differential equation
- Spatial discretization - finite difference scheme
- Temporal discretization - explicit Euler method

$$y_{i,n+1} = y_{i,n} + \Delta t v_{i,n}$$

$$v_{i,n+1} = \frac{\Delta t}{\rho_w}\left[\frac{y_{i+1,n} - 2y_{i,n} + y_{i-1,n}}{\Delta z^2} - \rho_w g + (IB_x)_i\right]$$



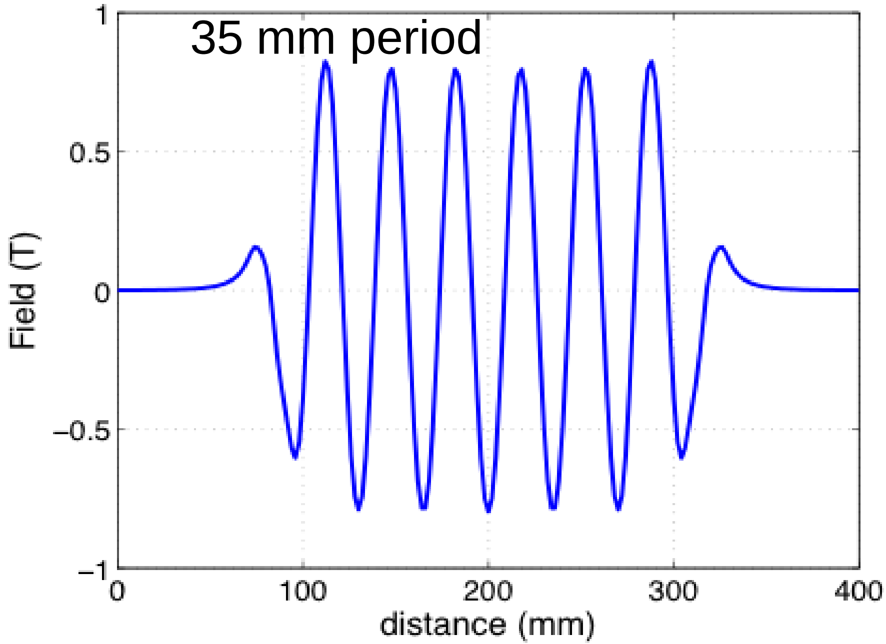


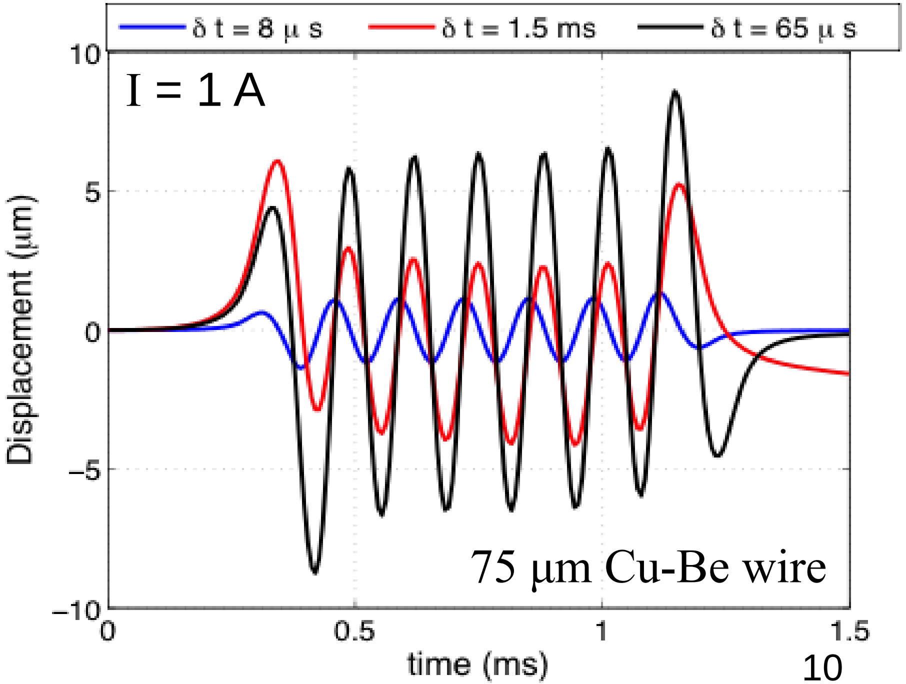
Pulsed Wire Parameter Trends

- The displacement is proportional to the current amplitude
- An increase in the pulse length increases the displacement for short pulse lengths
- Increase in tension
  - Decreases displacement
  - Decreases sag
  - Increases wave speed
- Increase in wire density (increase in diameter)
  - Decreases displacement
  - Increases sag
  - Decreases wave speed
- The wave speed and sag can be kept constant by scaling the tension to keep the stress in the wire constant (for the same wire material)
  - Stress scales with the inverse of area
  - Wire density scales with the wire area

Pulsed Wire Calculations

- Analytical calculations of displacement as a function of time for various pulse time lengths
- The maximum displacement occurs when the product of pulse time length and wave speed is equal to half of the length of a period
- May be best to choose the pulse time length to maximize signal to noise ratio
- Field can be calculated by solving the inverse problem with a parameterized field profile





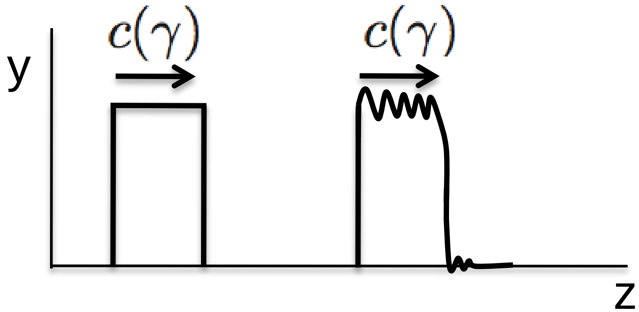
Pulsed Wire Simulation

5 μs pulse

1.5 ms pulse

Dispersion

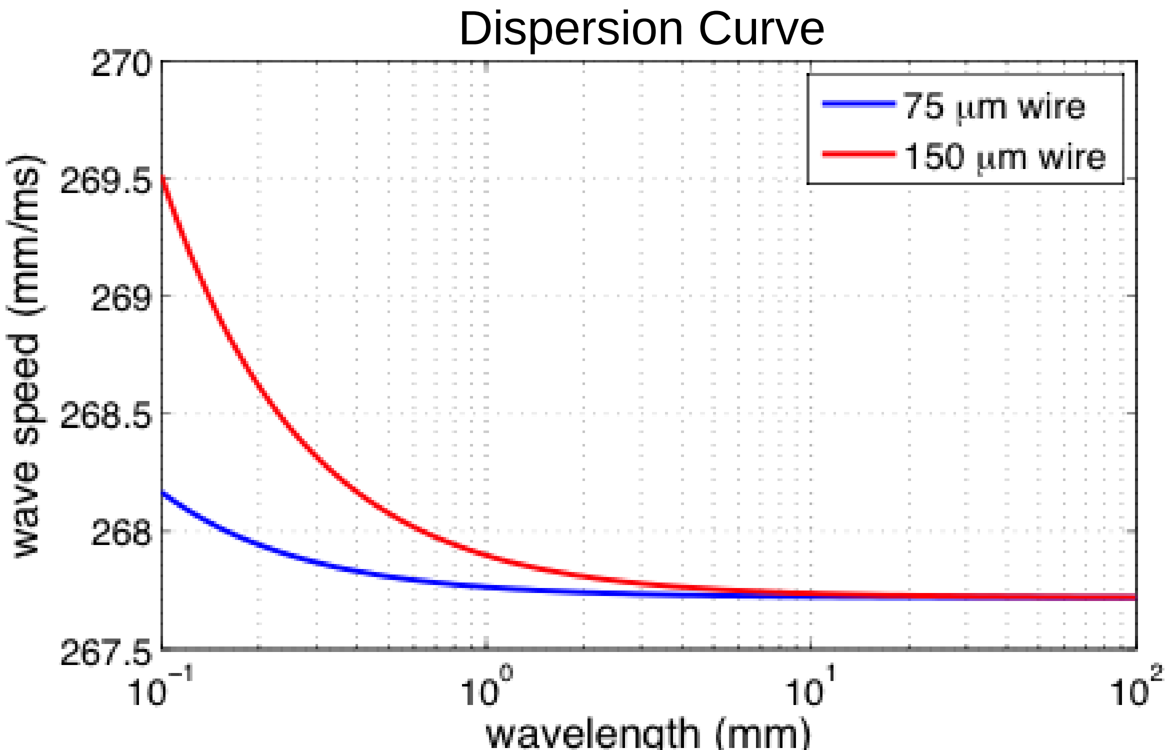
- The flexural rigidity of the wire leads to dispersive behavior
- Thin wires with lower flexural rigidity are less susceptible to dispersion
- Dispersive behavior can be predicted using Euler Bernoulli theory for bending of thin rods
- Theory looses accuracy at short wavelengths where the beam acts as a three-dimensional solid (shear effects become important)



$$T\frac{\partial^2 y}{\partial z^2} - EI\frac{\partial^4 y}{\partial z^4} = \rho A\frac{\partial^2 y}{\partial t^2}$$

$$y = Ae^{i\gamma(z-ct)}$$

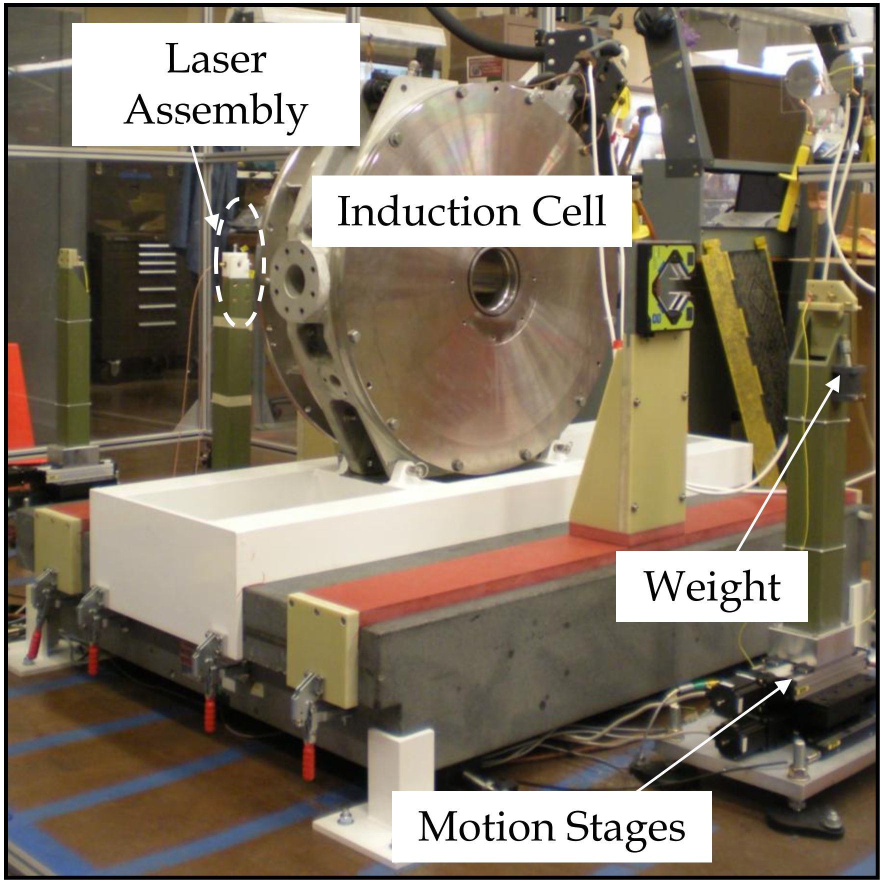
$$c = c_0\sqrt{1+\frac{EI}{T}\gamma^2}$$



Characteristics of Pulsed Wire Measurements of Undulators

- Pulse time length can be chosen to approximate the first and second field integrals or for maximum displacement
- Large transverse field (~ 1 T)
- Small field integrals for short period undulators
- Dispersive effects can be minimized by using thin wires
- Sensitivity to external perturbations and wire imperfections increases with smaller wires
- Measurement system parameters that can be optimized
  - Pulse time length
  - Extraction method for magnetic field (or integrals) from displacement data
  - Wire diameter (dispersion and wire sag)
  - Tension (dispersion and wire sag)

Measurement Bench



Measurement Bench Prototype:

-Wire: 75 μm Be-Cu

- Length = 250 mm

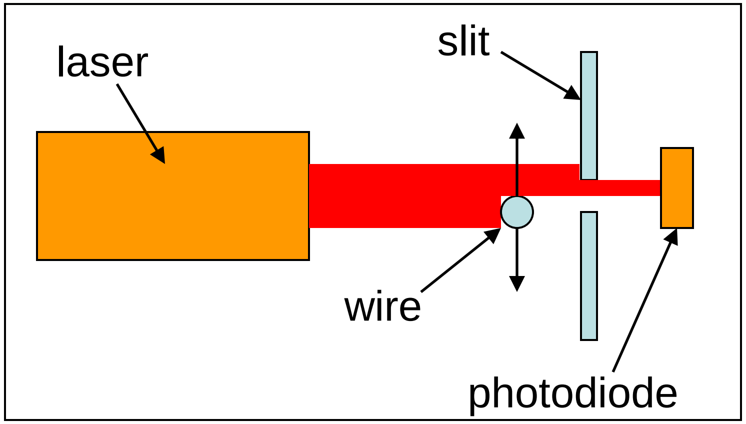
- Weight = 380 g

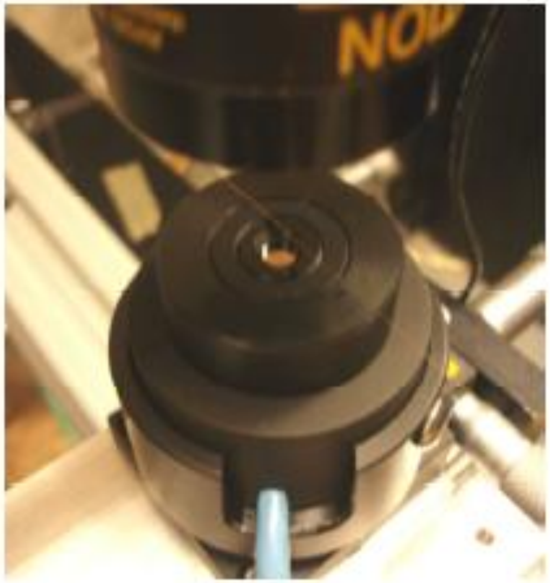
- Wire pulser: $I_w < 7$ A

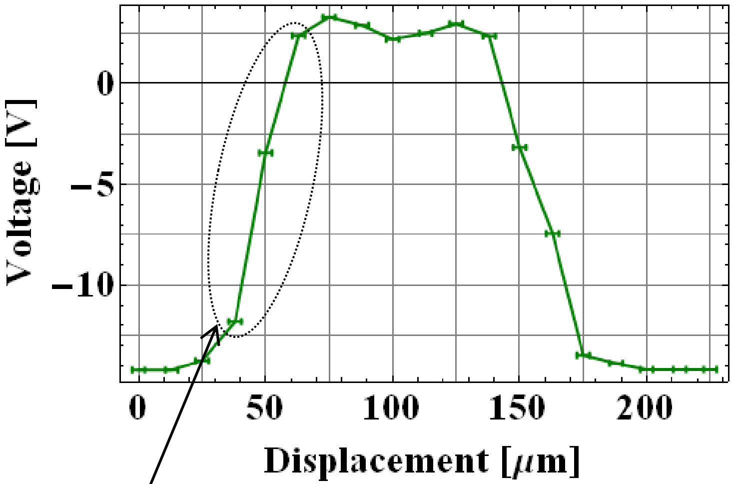
- Adjustable wire position

- Laser micrometers to determine wire position

Laser Assembly

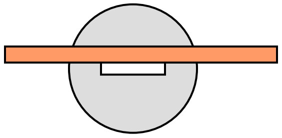




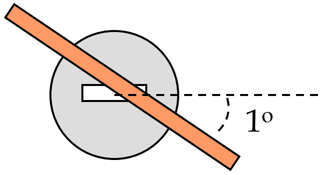


Region of interest:
500 mV/µm (mean value)

Sensitivity depends on angle between slit and wire

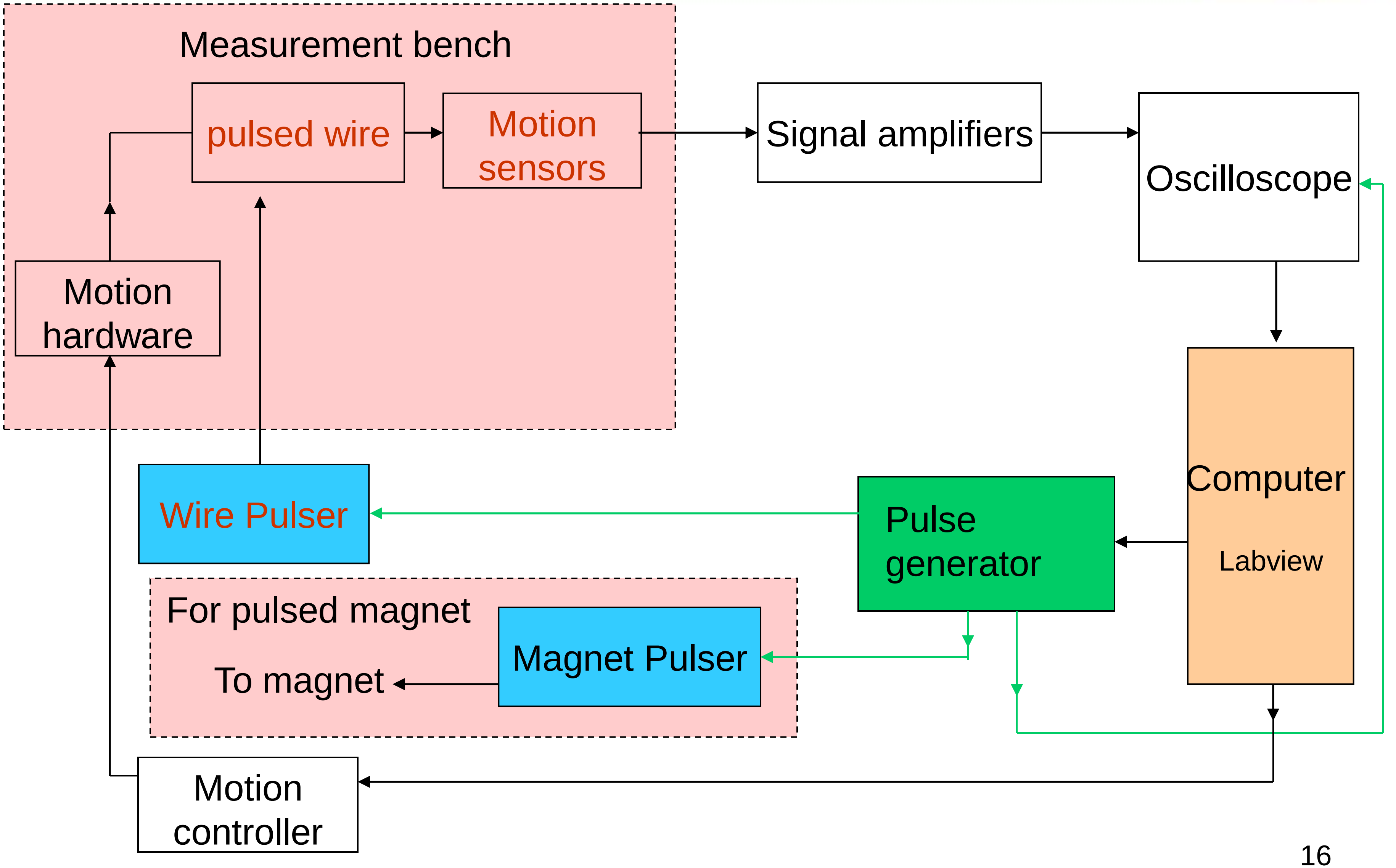


500 mV/µm



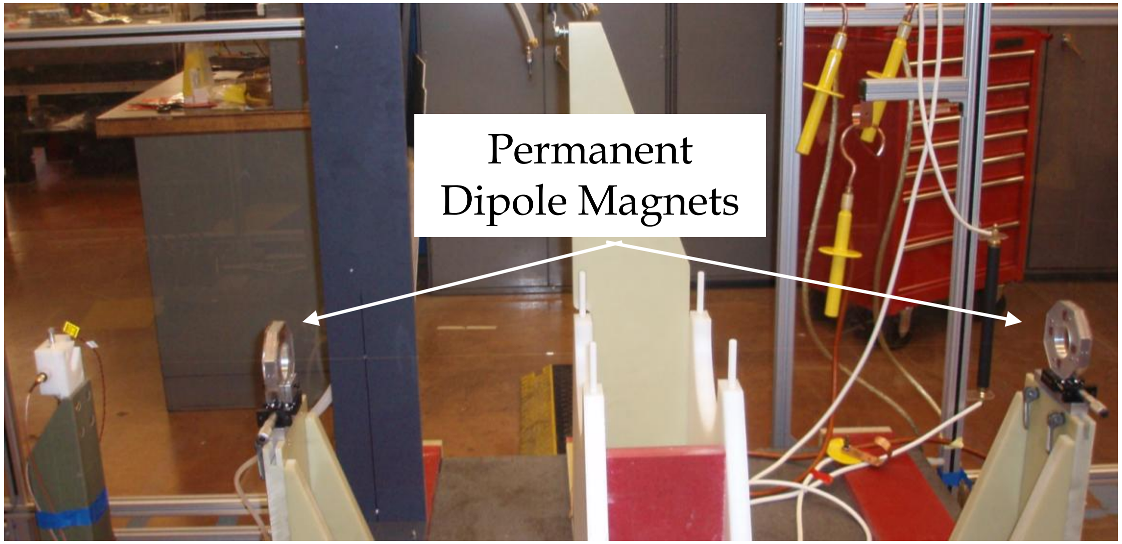
415 mV/µm

Diagram of Acquisition / Control System

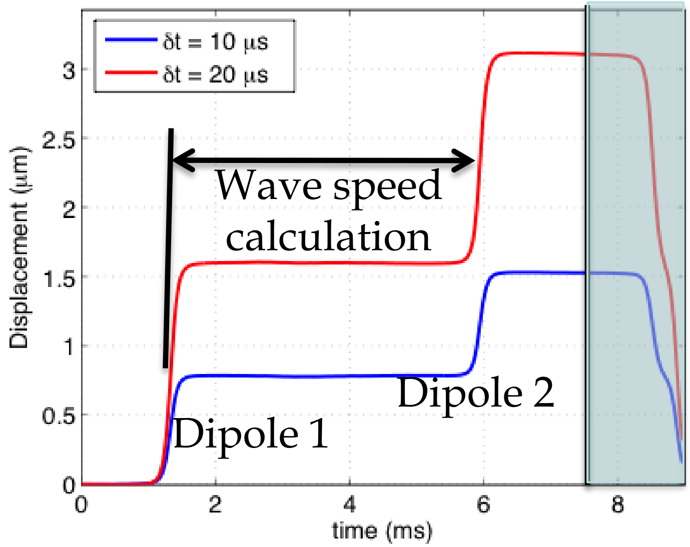


Permanent Dipole Magnet Measurements

- Permanent dipole magnets used to characterize the pulsed wire system
- Distance between magnets used to determine the wave speed
- Signal is averaged over many samples to reduce the effect of environmental noise (100 samples for data below)
- Offset motion from background magnetic field (e.g. Earth's magnetic field) is removed by pulsing the wire with the magnets removed from the measurement setup

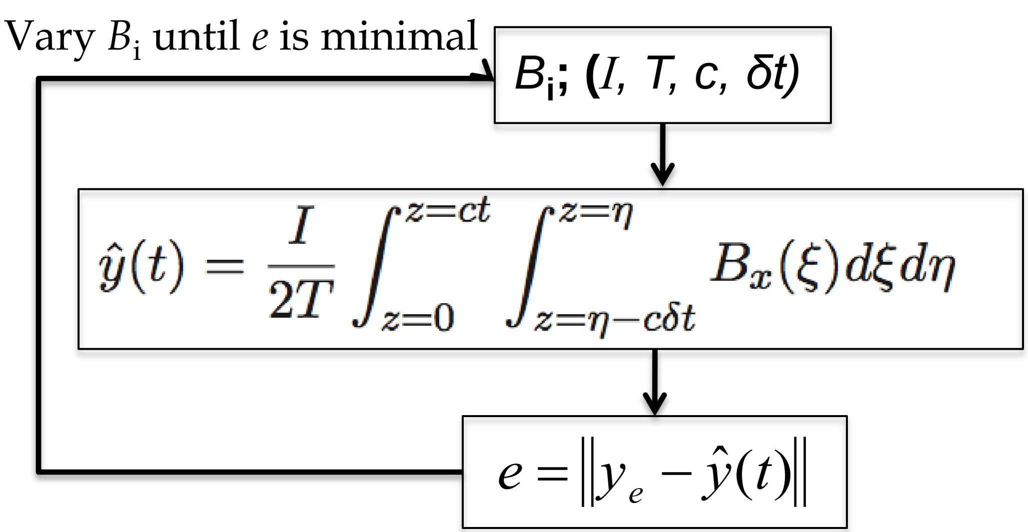


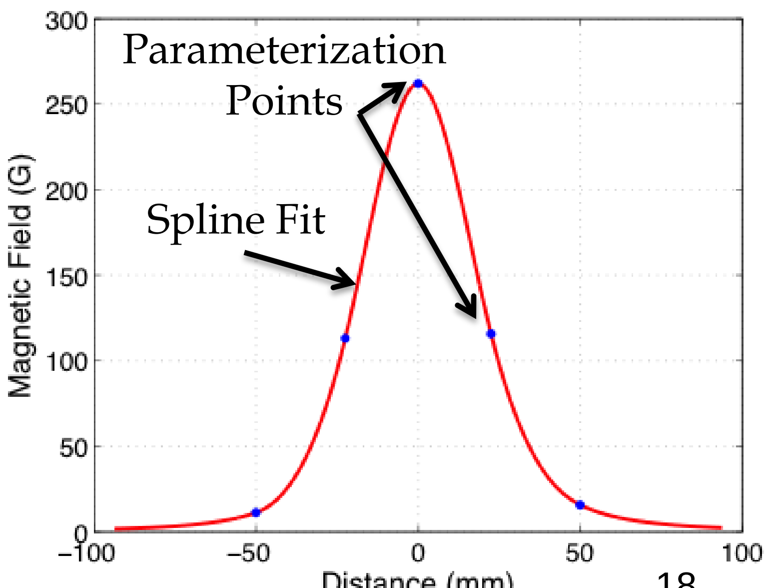
Wire Motion Measurement



Parameterized Field Inverse Calculation

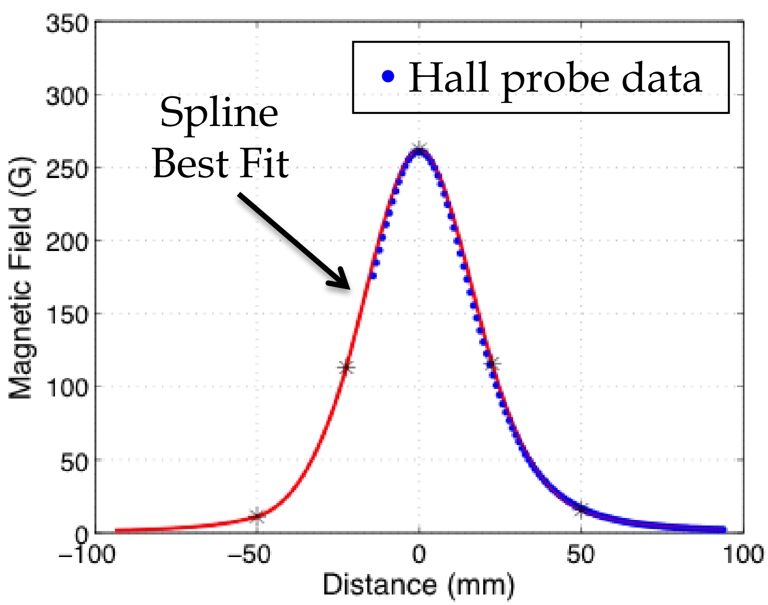
- For short pulses the field can be determined by taking the derivative of the displacement signal
  - Possible low signal to noise ratio (displacement is proportional to pulse width)
  - High frequency noise is amplified with this approach
  - Data smoothing can be performed but care must be taken not to smooth away the “real” signal
- Magnetic field can be parameterized by control points
  - Displacement is calculated from analytical expression as a function of the various parameter
  - Control points that minimize the error between the measured and calculated data are determined

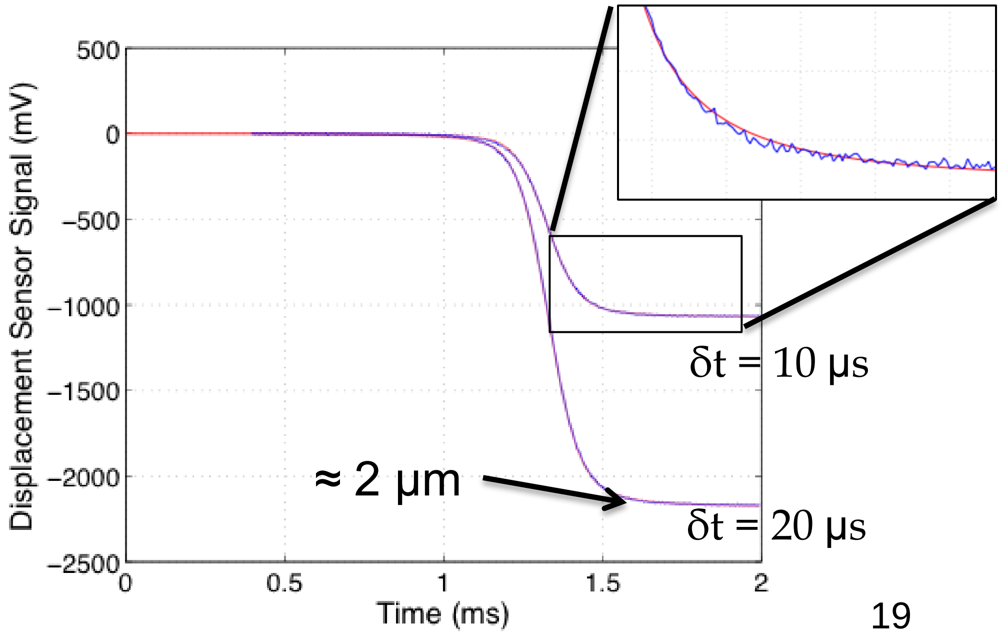




Parameterized Field Example

- Scaling from motion sensor sensitivity chosen to match peak field value
  - Sensor sensitivity is difficult to measure precisely due to imprecision and inaccuracy in sensor motion (achieved with stepper motor driven stage)
  - Determined value lies within the uncertainty region of the directly measured sensor sensitivity
- For large motions accurate scaling can be affected by the motion sensor non-linearity
- Calculated field compared to partial Hall probe mapping
- Best fit spline field has a slightly wider profile than the Hall probe measurement and a slight asymmetry



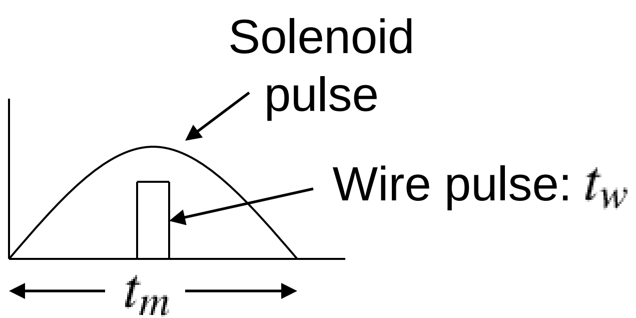


Example: Pulsed Solenoid Alignment

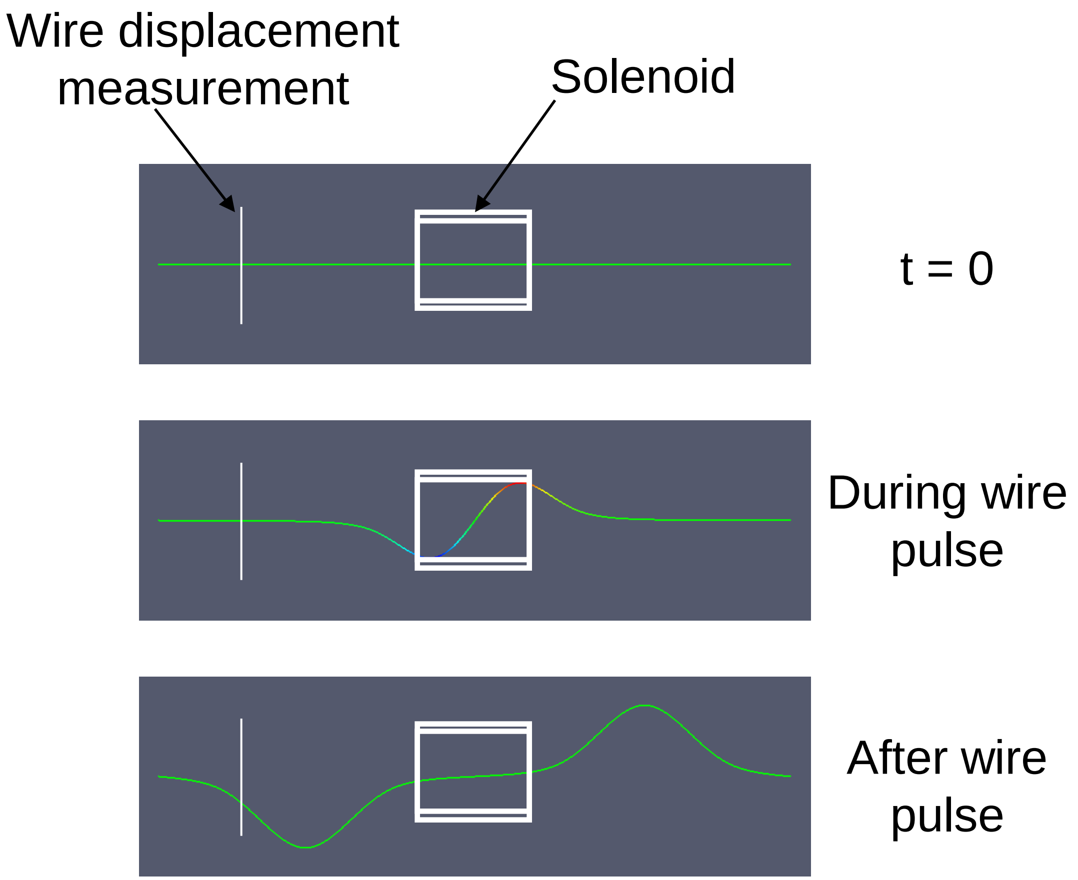
- Alignment of pulsed solenoids for an ion induction accelerator
- Peak axial magnetic field is approximately 2.5 T with a 5 ms pulse
- Magnetic axis misalignments lead to corkscrew deformation of the beam and reduced intensity
- Pulsed wire technique was chosen after investigating several options
  - Simplicity in positioning the wire
  - Wire defines an axis directly

Pulsed Wire Measurements on Pulsed Solenoids

- Ion beam moves across the solenoid in a 10 ns window at the peak of the pulse
- Axis location can vary as a function of time due to eddy currents
- Need a short wire pulse at the peak of the solenoid pulse

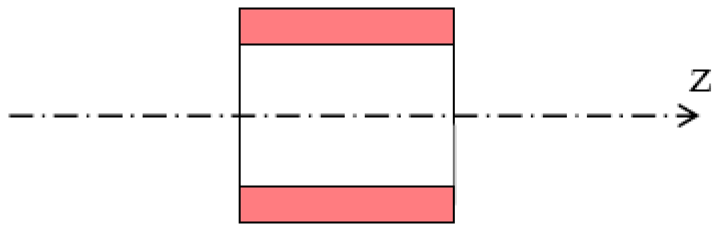


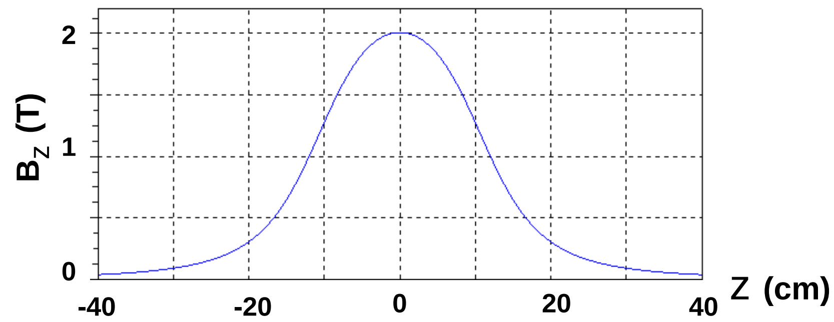
t_m ~ 5 ms



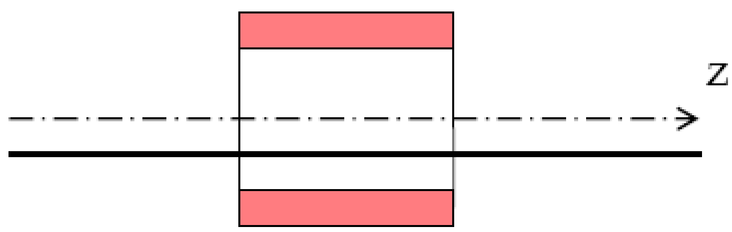
Ideal Solenoid Field Approximation

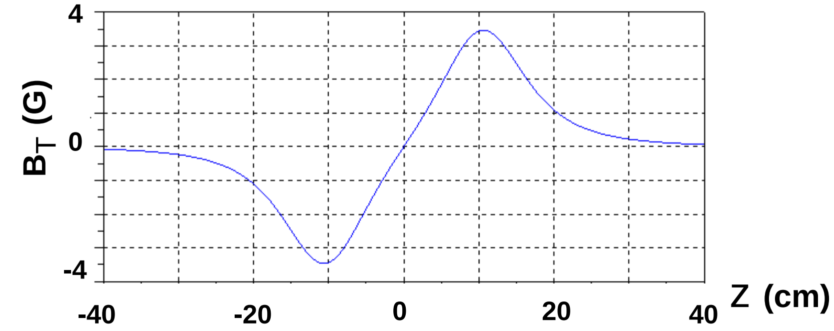
On axis axial field



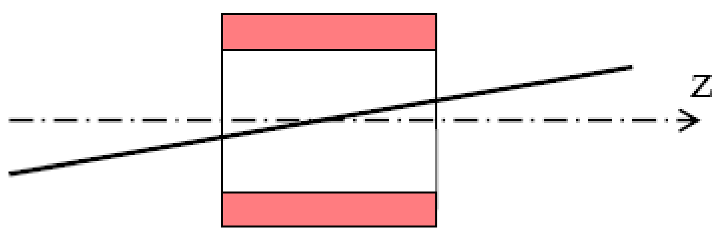


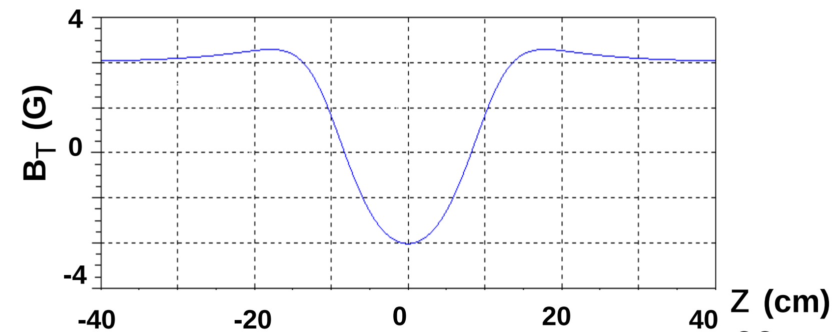
Off axis transverse field





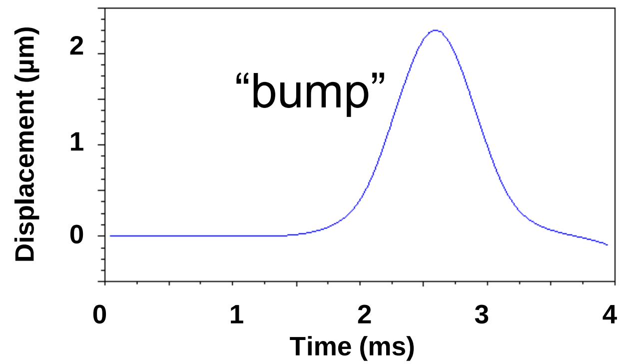
Tilted axis transverse field



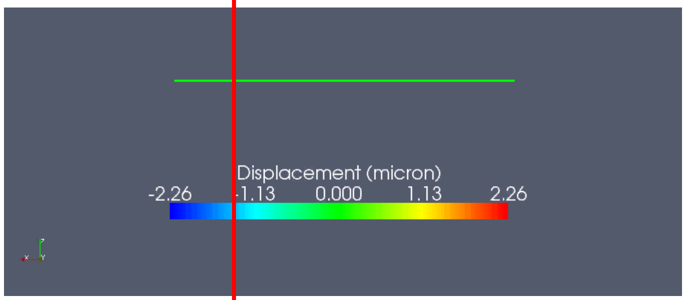


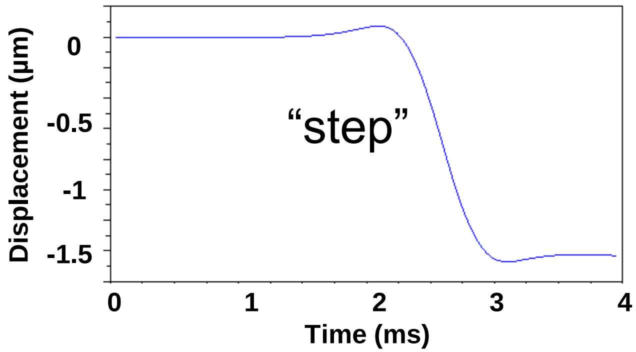
Pulsed Wire Simulations

Measured displacement at a specific point

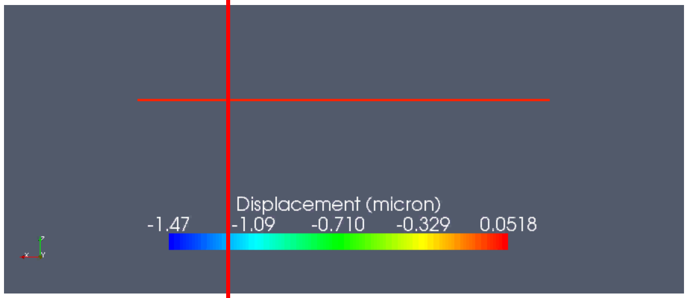


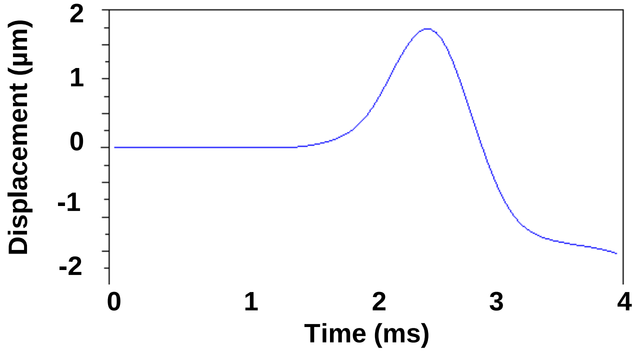
50 μm offset





100 μrad tilt

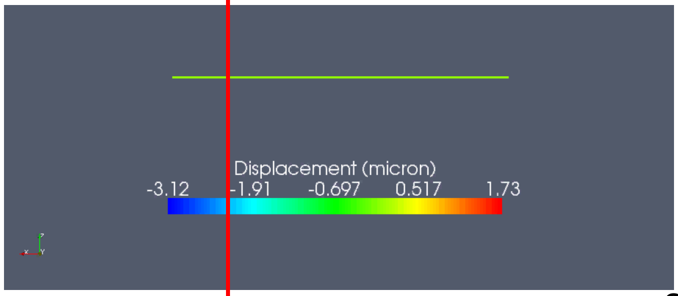




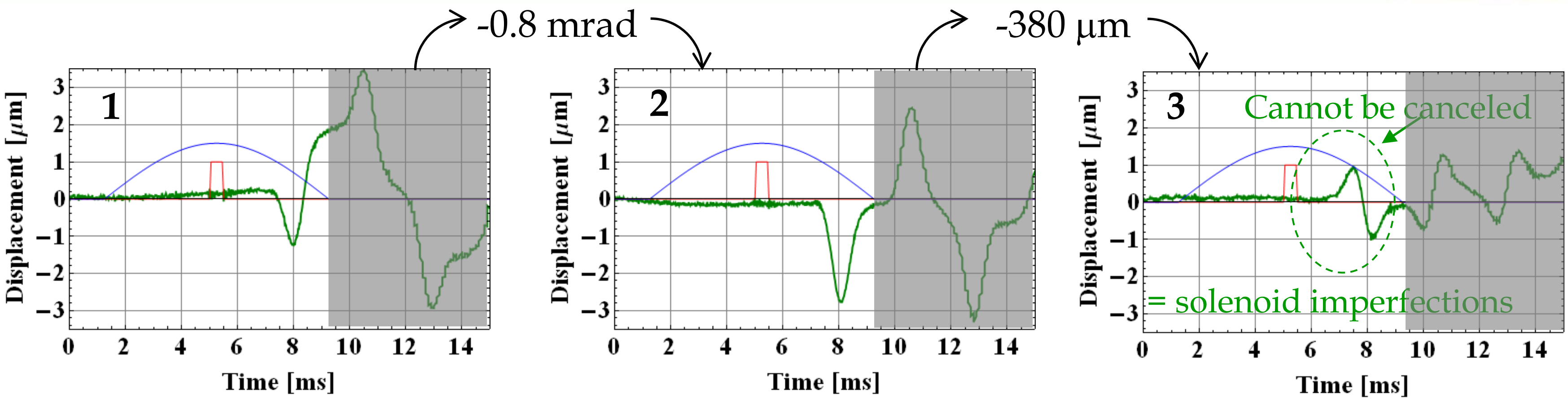
50 μm offset

AND

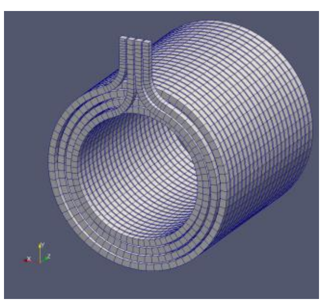
100 μrad tilt

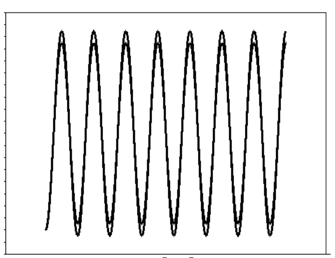


Locating the Axis



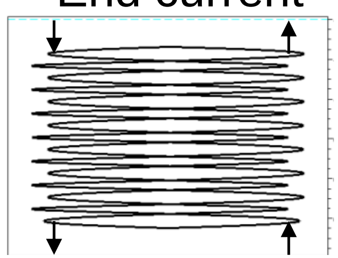
Geometric Model with Realistic Windings





Side

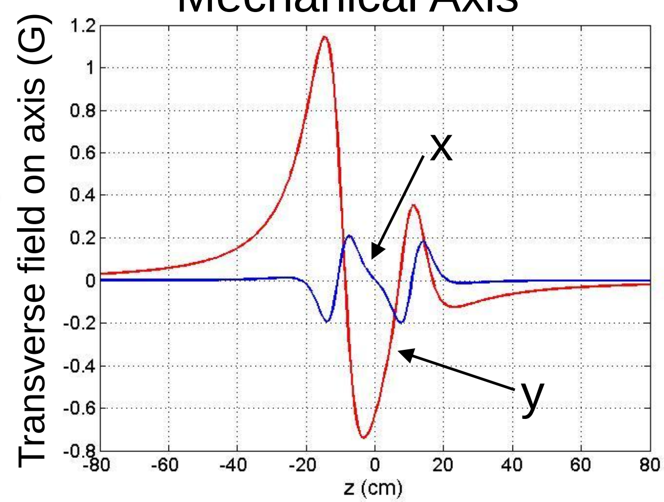
End current



Top

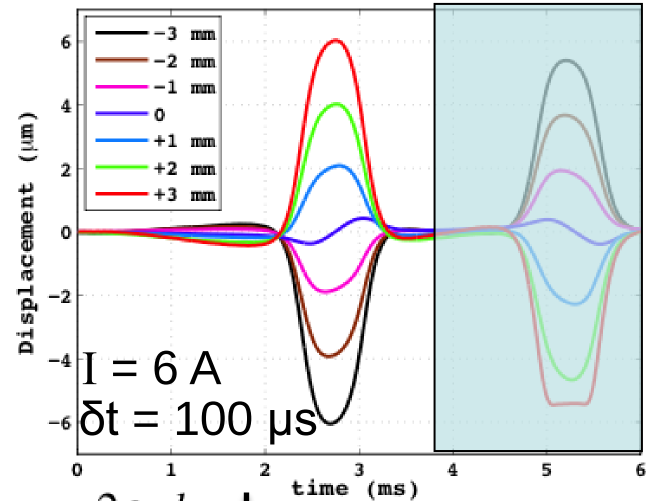
Biot Savart Calculation

Calculated Field on Mechanical Axis

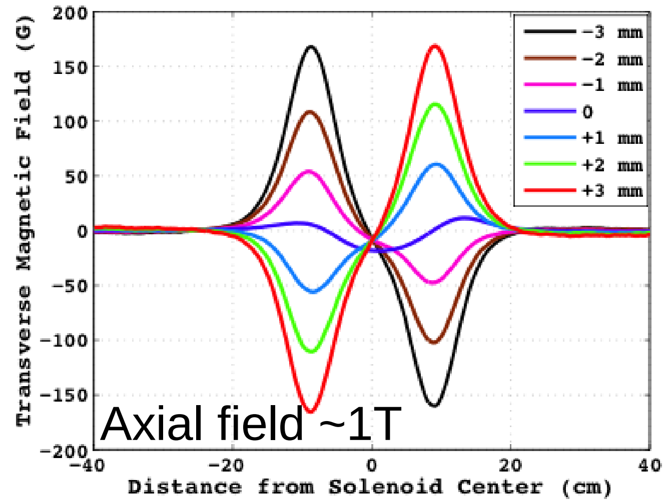


Estimating the Transverse Field from Wire Motion Measurements

- Field estimate can be determined from the transverse wire velocity
- Assumptions
  - Long wire (no reflections)
  - Short wire pulse
- Position is determined using the wave speed
- Error introduced by the finite length of the wire pulse
- Error is small even with a relatively long pulse (100 μs) since the magnetic field is varying slowly over a large distance
- Slight data smoothing is used to reduce high frequency noise
- 3D field mapping can be performed
  - Aligned wire offsets for transverse field
  - Tilted wire offsets for projected axial field



$$B_y = \frac{2\rho}{It_w}\frac{dx}{dt}$$



Conclusions

- Pulsed wire technique is being investigated for the measurement of undulators where limited space and cold temperatures place constraints on the possible measurement techniques
- The accuracy of the method must be improved to be a competitive magnet measurement choice

Main Issues to be addressed

- Motion sensor linearity or accurate calibration
  - Laser stability
  - Investigate use of optical equipment to improve sensor non-linearity
- Sag (use of high-strength wire)
- Background noise reduction
- Accurate wave speed calculation
- Accurate measurement of wire current pulse
  - Investigate the effect of rise time in wire pulse
  - Numerical simulations can be used
- Magnetic field extraction
  - Introduce physically meaningful parameterization
- Dispersion
  - May be an issue for short period undulators
  - Investigate the use of thin wires